Retail Discrimination in Search Markets*

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Friday 24$^{\text{th}}$ August, 2018

Abstract

This paper analyses the incentives of manufacturers to discriminate between ex ante symmetric retailers competing for consumers with different search cost. By discriminating, a manufacturer indirectly screens searching consumers, creates more retail competition, increases its profits, but lowers consumer welfare. Low-cost retailers sell to a disproportionate share of low search cost consumers, providing strong incentives to compete; high-cost retailers also lower margins given their smaller customer base. For wholesale price discrimination to be an equilibrium outcome, some form of commitment is necessary. Legislation requiring sales at the recommended retail price serves as such a commitment device, making consumers worse off.

JEL Classification: D40; D83; L13
Keywords: Vertical Relations, Consumer Search, Double Marginalization, Wholesale Price Discrimination.

*We thank audiences at Bonn, EARIE 2017, the VIII Workshop on Consumer Search and Switching Cost, the Industrial Organization section of the Verein für SozialPolitik and VGSE micro seminars at the University of Vienna, for helpful comments. In particular, we would like to thank Mark Armstrong, Heski Bar-Isaac, Stephan Bühler, Rahul Deb, Arthur Fishman, Daniel Garcia, Stephan Lauermann, Dmitry Lubensky, Eeva Mauring, Martin Obradovits, Alexei Parakhonyak and Anton Sobolev. We acknowledge financial support from the Austrian Science Foundation FWF, under project number P 27995-G27.

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1 Introduction

Wholesale price discrimination is customary in many important markets. In fact, most of the antitrust cases involving price discrimination seem to involve B2B relations between manufacturers (or suppliers) and retailers rather than B2C relations.[1] The Robinson-Patman Act,[2] the main piece of legislation in the U.S.A. dealing with wholesale price discrimination, considers these practices to be illegal if their effect "may be to lessen competition". Empirical studies dealing with wholesale price discrimination include Villas-Boas [2009] (on the relation between coffee manufacturers and supermarkets in Germany) and Hastings [2009] (on gasoline markets in the U.S.A.). Many of these markets are characterized by significant informational frictions on the side of consumers. However, the literature on wholesale price discrimination has largely concentrated on markets where consumers are fully informed about prices.

In this paper, we consider vertical industries with consumer search and provide a new theory of why manufacturers with significant market power may engage in wholesale price discrimination. We show that by setting different wholesale prices to different retailers, a manufacturer stimulates search, screens consumers according to their search costs and creates a more competitive retail market, which boosts her profits. Despite creating a more competitive retail market, we show that consumers are actually worse off as the manufacturer, faced with a more inelastic demand, sets higher wholesale prices. Thus, we provide a new perspective on the economic rationale behind the Robinson-Patman Act, as wholesale price discrimination lowers welfare even though it strengthens (retail) competition.

To gain insight suppose that under wholesale price discrimination, a manufacturer charges a low wholesale price to some retailers and a high wholesale price to others, resulting in low and high retail prices in the downstream market. Expecting some price dispersion, without knowing which retailer charges lower prices, consumers with different search cost will follow different search paths after their initial search: observing a high retail price at their first search low search cost consumers continue to search, while others will buy immediately. As a consequence, retailers do not face the same composition of search costs among their costumers: the demand of low cost retailers consists of a relatively larger share of low search cost consumers and as these consumers continue searching if

[1] See, e.g., the claim of Games People Play, a retailer for golf equipment in the US, against Nike, ruled by the federal district court in Beaumont, Texas in February 2015 (Games People Play, Inc. v. Nike, Inc.; case number 1:14-CV-321), or, earlier cases such as the decision on the European sugar industry in 1973 where the Commission ruled that, "the granting of a rebate which does not depend on the amount bought [...] is an unjustifiable discrimination [...]" (Recital II-E-1 of Commission decision 73/109/EC), or the the Michelin I judgement where the European Commission in 1981 contested the alleged discriminatory nature of wholesale prices (Recital 42 of Commission decision 81/969/EEC).

[2] The European Union's Article 102 (c) of the Treaty seems to be more restrictive and forbids dominant firms to apply "dissimilar conditions to equivalent transactions with other trading parties, thereby placing them at a competitive disadvantage".

they expect lower prices elsewhere, this will induces more competition between low cost retailers. In addition, a high cost retailer may also lower margins compared to uniform wholesale (and retail) pricing as they have a smaller base of consumers and marginally raising their price will lead to a proportionally large share of consumers leaving the firm. Thus, both low and high cost retailers may have lower margins under wholesale price discrimination. As lower retail margins, *ceteris paribus* increase manufacturer profit, the manufacturer may positively consider to engage in wholesale price discrimination.

For the same wholesale prices, consumers benefit from lower retail margins, but whether consumers *de facto* benefit from wholesale price discrimination depends on the wholesale prices the manufacturer chooses. We show that whether or not the manufacturer actually engages in wholesale price discrimination and which prices she chooses depends on the extent to which she can commit to the wholesale prices set. In industries that have relatively stable cost and demand patterns, a manufacturer may commit by setting long-term wholesale contracts. Under commitment, we show that wholesale price discrimination increases manufacturer profits and reduces retail profits and consumer surplus. As retailers are faced with more elastic demands, retail prices will react less to wholesale price changes under wholesale price discrimination compared to uniform pricing, creating a more inelastic demand for the manufacturer and providing her with an incentive to increase wholesale prices. In other industries, it may be difficult to commit to charging different retailers different prices. Without commitment, we show that wholesale price discrimination cannot be sustained as an equilibrium outcome. If the manufacturer cannot commit then wholesale price discrimination can only be an equilibrium if the manufacturer makes identical profits across retailers. This equal profit condition is inconsistent with other equilibrium conditions that the wholesale prices have to satisfy.

For markets where the manufacturer cannot commit to wholesale prices, we look at the effect of recommended retail prices (RRPs) and the regulation imposed by the U.S. Code of Federal Regulations, used by the Federal Trade Commission, which requires that at least some sales have to take place at list prices.[3] We argue that this regulation effectively facilitates manufacturers to partially commit and engage in wholesale price discrimination as follows. The manufacturer may announce the retail price that the high cost retailer finds optimal to charge as the RRP. Given the Code and after such an announcement, the manufacturer should make sure that at least some products are sold at this price and thus she is not free to deviate and sell to all retailers at the lower wholesale price that generates more profits. We show that once this possible deviation is eliminated, wholesale price discrimination can be sustained as an equilibrium outcome and that, relative to uniform pricing, the average wholesale and retail prices increase, increasing manufacturer profits, but decreasing retailers' profits and consumer welfare.

[3] RRPs are non-binding recommendations of manufacturers at which prices retailers should sell their product. As retailers are free to deviate from the recommendation, an important question is whether these recommendations affect market behaviour and if so how.

In summary, we show that a manufacturer may price discriminate between ex ante symmetric intermediaries or retailers, to exert a competitive effect on a market in which she is not active herself. Wholesale price discrimination effectively is a mechanism to (indirectly) screen searching consumers in vertical markets. Essential to our theory is that the retail market is characterized by consumer search and that the manufacturer can (partially) commit to wholesale prices either directly or indirectly via the regulations that apply to the use of RRPs. Wholesale price discrimination ensures that there is price dispersion at the retail level stimulating some low search cost consumers to actively search beyond the first firm. To focus on the new elements of our theory, we consider a setting where all retailers are *ex ante symmetric*[4] and all consumers have *identical demand*.[5] Apart from these main results, we also make methodological contributions to the literature on consumer search in vertical markets.

There are several branches of the literature to which this paper contributes. First, the starting point of seminal papers in the literature on price discrimination in intermediate goods markets (Katz [1987], DeGraba [1990] and Yoshida [2000]) is that downstream firms differ in their efficiency levels. A monopolist manufacturer who is unconstrained by possible demand substitution may choose to charge higher wholesale prices to more efficient firms, decreasing total surplus relative to uniform wholesale pricing. Inderst and Valleti [2009] show that a ban on discrimination may have negative effects if the assumption of an unconstrained manufacturer is relaxed. The novelty of our paper is that in many markets consumers must engage in costly search to get to know market prices. By taking into consideration information frictions regarding retail prices, a manufacturer may purposefully create asymmetries between retailers that are ex ante symmetric.

Second, there is a recent literature on vertically related industries with consumer search. Janssen and Shelegia [2015] show that markets can be quite inefficient if consumers search sequentially while not observing the wholesale arrangement between the manufacturer and retailers. Importantly, and in contrast to our paper, the manufacturer always sets the same wholesale price to all retailers and retailers know this. In addition, unlike Janssen and Shelegia [2015], equilibrium existence is not an important issue in our paper due to the general (continuous) search cost distribution we consider. Garcia,

[4]The Robinson-Patman Act was enacted with the aim of protecting "mom and pop" stores from the rise of chain supermarkets that were rewarded with better terms by manufacturers simply because of their larger purchasing abilities. However, when retailers differ, either in terms of efficiency or size, price discriminating suppliers may justify their practice by pointing at cost differences (See: https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/price-discrimination-robinson-patman). The Robinson-Patman Act is nowadays often applied in settings where the buying firms are ex ante symmetric, making a discriminating manufacturer's cost defence improbable. This is the situation we study in this paper.

[5]Bergstrom and Varian [1985] and Salant and Shaffer [1999] show that unequal treatment of identical firms in a Cournot setting may lead to an increase in total surplus. Unlike these papers, we show in a very different setting that an upstream manufacturer wants to treat her retailers differently and that this leads to negative welfare effects.

Honda, and Janssen [2017] show that the inefficiency of vertical markets with consumer search continues to hold if there are many manufacturers and retailers engage in sequential search among these manufacturers. Garcia and Janssen [2017] focus on the optimal correlation structure for wholesale prices. In contrast, we focus on how wholesale price discrimination affects the search cost composition of demand for different retailers and how this affects the competitiveness of the retail market. Asker and Bar-Isaac [2016] study different potential roles of minimum advertised prices (MAPs) with price discrimination as one of them. The rationale for wholesale price discrimination in their paper is close to the traditional role for price discrimination in extracting surplus from consumers with different valuations. In contrast, in our model consumers have identical valuations and wholesale price discrimination is a way to screen consumers with different search cost. We therefore have a purely informational story of price discrimination.

Third, another recent literature explains how non-binding RRPs may affect market behaviour. Buehler and Gärtner [2013] and Lubensky [2017] use a framework where RRPs are used by the manufacturer to signal production cost. Buehler and Gärtner [2013] see RRPs as communication devices between a manufacturer and her retailers with RRPs as part of a relational contract. Lubensky [2017] shows that a manufacturer can use RRPs to signal his production cost to searching consumers. In contrast to these papers, uncertainty concerning manufacturer cost does not play a role in our setting. We show that regulation requiring that at least some sales are made at RRPs can serve as a commitment mechanism that enables a monopolist manufacturer to engage in wholesale price discrimination.

Fourth, there is a small literature on consumer search and price discrimination. In a market where the demand of high search cost consumers is less price sensitive than the demand of low search cost consumers, Salop [1977] shows that a monopolist who directly sells to consumers may engage in price discrimination: as low search cost consumers continue to search if they first encounter a high price, higher prices attract a disproportionally large fraction of consumers with higher search cost, who (by assumption) are also less price-sensitive. Unlike our purely informational theory of price discrimination, Salop [1977] follows the classical view of price discrimination as distinguishing between consumers with different valuations. In addition, his argument is based on the assumption that the monopolist retailer is committed to charging prices according to a price distribution and that any deviation from this distribution is observed by consumers. It is difficult to see, however, how consumers may observe a price distribution, while maintaining the assumption underlying the search cost literature that the consumer does not know the prices firms set. By studying a vertical supply chain, our paper, in contrast, can make a distinction between a manufacturer committing to wholesale prices to retailers, while consumers search for retail prices. Fabra and Reguant [2018] focus on markets with small and large buyers where large buyers have more incentives to search making firms compete more strongly for them. Again, and in contrast to our paper, differences in demand push firms to price discriminate in their paper. Likewise, differences in consumers' valuations (related to differences in the cost of studying products), and not differences in search (browsing) costs, lead to low valuation consumers paying lower prices also in the model studied by Heidhues, Johannes, and Köszegi [2018].

Finally, while most papers in the search literature assume at most two different levels of search cost (see, e.g., Stahl [1989]), there do exist some papers that consider more general forms of heterogeneity in consumers' search costs, such as Stahl [1996], Chen and Zhang [2011] and Moraga-González, Sándor, and Wildenbeest [2017]. In contrast to these papers, however, we focus on vertically related industry structures and this paper is the first to consider general forms of search cost heterogeneity in such settings.

The main body of the paper analyzes markets with linear wholesale pricing given that two-part tariffs, despite their theoretical appeal, are not often used in actual business transactions. Blair and Lafontaine [2015] state that, even in situations when two-part tariffs are adopted, the fixed component seems to be a relatively small part of the overall payment between firms (see, also, Kaufmann and Lafontaine [1994]). Differences in demand expectations, in risk attitude, the possibility of ex-post opportunism by the supplier and wealth constraints by the retailers are mentioned among reasons why two-part tariffs are not often implemented in actual transactions. In an online Appendix, we show that our analysis is robust to manufacturers setting a fixed fee extracting part, but not all, of the retail profits.

The remainder of this paper is organized as follows. In the next section, we present the details of the model we consider. The impact of wholesale price discrimination on the retail market is discussed in Section 3. Section 4 discusses the commitment case under both uniform pricing and wholesale price discrimination while the non-commitment case is discussed in Section 5. In both sections, we first provide analytic results for the case where search costs vanish, followed by numerical analysis for the linear demand case that allow us to show the robustness of our theoretical results. In Section 6, we analyse the implications of imposing that some sales take place at the list price, as the Code of Federal Regulations demands. Finally, Section 7 concludes, while proofs are in the Appendix. An online Appendix elaborates on some technical arguments and discusses two extensions.

2 The Model

We focus on a vertically related industry with a monopolist manufacturer in the upstream market supplying a homogeneous product to $N \geq 3$ retailers.[6] The manufacturer's production costs are normalized to zero. In principle, the manufacturer can charge a different wholesale price w_i to every retailer, so that formally the manufacturer's strategy is a tuple $(w_1, w_2, ..., w_N)$.[7] We will focus on two types of equilibria: (i) in a uniform pricing

[6]To study the effects of wholesale price discrimination, it is important there are at least two retailers that get the lowest wholesale price so we need at least three retailers in the downstream market.

[7]In the main body of the paper we assume that the manufacturer charges linear whiolesale prices. In an online Appendix we show that the results are robust to non-linear prices in case the manufacturer

equilibrium the manufacturer chooses $w_i = w^*$, whereas in an equilibrium with price discrimination the manufacturer chooses two prices w_L^* and w_H^*, with $w_L^* < w_H^*$, and charges some retailers the low and others the high wholesale price.[8] Retailers take their wholesale price as given and do not face other costs except for the wholesale price paid to the manufacturer for each unit they sell.

There is a unit mass of consumers, each demanding $D(p)$ units of the good if they buy at price p. We make standard assumptions on the demand function so that it is well-behaved. In particular, there exists a $\overline{p}$ such that $D(p) = 0$ for all $p \geq \overline{p}$ and the demand function is continuously differentiable and downward sloping whenever demand is strictly positive, *i.e.*, $D'(p) < 0$ for all $0 \leq p < \overline{p}$. For every $w \geq 0$, the retail monopoly price, denoted by $p^M(w)$ is uniquely defined by $D'(p^M(w))(p^M(w) - w) + D(p^M(w)) = 0$ and $D''(p)(p-w)+2D'(p) < 0$. Note that for $w = 0$, this condition gives that the profit function of an integrated monopolist is concave. We denote by $p^M(w^M)$ the double marginalization retail price, which arises in case there would be a monopoly at both levels of the supply chain. In numerical examples, we consider demand to be linear, $D(p) = 1 - p$.

In order to observe prices consumers have to engage in costly sequential search with perfect recall. Consumers differ in their search cost s. Search costs are distributed on the interval $[0, \overline{s}]$ according to the distribution function $G(s)$, with $G(0) = 0$. We denote by $g(s)$ the density of the search cost distribution, with $g(s) > 0$ for all $s \in [0, \overline{s}]$ and a finite M such that $-M < g'(s) < M$. In numerical examples, we consider $G(s)$ to be uniformly distributed. As consumers are not informed about retail prices before they search, an equal share of consumers visits each retailer at the first search.[9]

For given expected wholesale prices and given their own wholesale price, an individual retailer i sets his retail price p_i, $i = 1, ..., N$. For given expected wholesale prices, consumers sequentially search for retail prices. The difference between the case where the manufacturer can and cannot commit is that in the latter case, but not in the former, the manufacturer may deviate from the wholesale prices consumers and other retailers expect without being noticed. The precise timing and the respective equilibrium notions are discussed in the relevant Sections.

3 The retail market

As explained in the Introduction, the main reason why a manufacturer may want to price discriminate between different retailers is to create a more competitive retail market. In this section, we explain in detail the mechanism by means of which this works and characterize the behaviour of consumers and retailers. As we will assume in the full

cannot fully expropriate the retailers' profits.

[8]The question whether the manufacturer can further increase profits by setting more than two different wholesale prices is left for future research.

[9]For most part of the analysis, it does not matter whether or not the first search is costly. We proceed assuming the first search is for free and do not consider the participation constraint of consumers.

equilibrium analysis that if a consumer observes an unexpected retail price, she believes that the retailer that she has visited has deviated and that all other retailers charge their equilibrium prices (passive beliefs), we will also do so in this Section. In case of wholesale price discrimination, when consumers expect different retail prices to prevail we consider in addition that if a consumer observes a price in a neighbourhood of one of the prices she expected, then the consumer believes that the deviation comes from a retailer that was expected to set a price that is closest to the observed price.

As a benchmark, consider first the case of uniform pricing where all retailers are expected to have the same wholesale price w^*. Let $p^*(w^*)$ denote the equilibrium price charged by all retailers (which is the retail price consumers expect). To determine the equilibrium retail price, we need to investigate how a retailer's demand depends on his own price, which in turn depends on how consumers' search behaviour reacts to a price deviation. If a consumer buys at a deviation price $\widetilde{p} > p^*$, he gets a surplus of $\int_{\widetilde{p}}^{\overline{p}} D(p)dp$. Under passive beliefs, a consumer with search cost s continues to search for the equilibrium price $p^*(w^*)$, if $s < \int_{p^*(w^*)}^{\overline{p}} D(p)dp - \int_{\widetilde{p}}^{\overline{p}} D(p)dp = \int_{p^*(w^*)}^{\widetilde{p}} D(p)dp$.

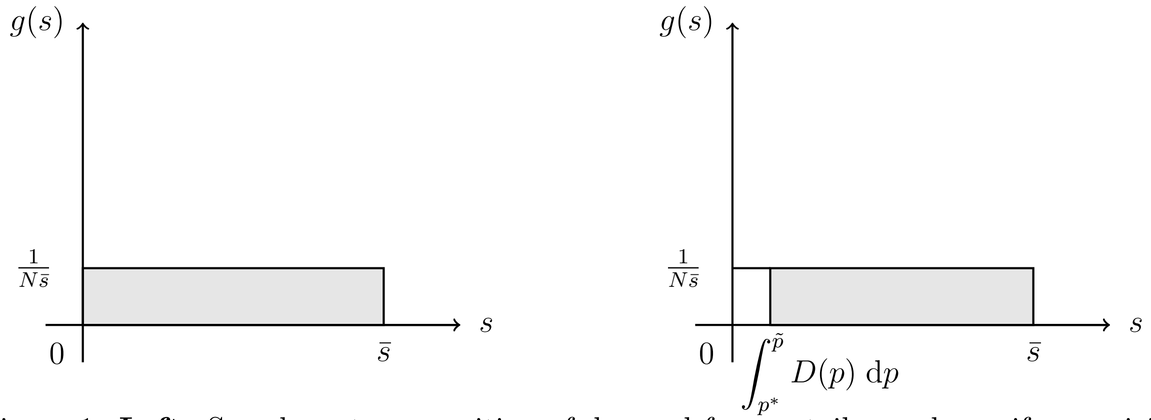


Figure 1: **Left:** Search cost composition of demand for a retailer under uniform pricing **Right:** Share of consumers that buy at the deviating retailer; where $s \sim U[0, \overline{s}]$.

Thus, of all consumers who visit a retailer deviating to a price $\widetilde{p} > p^*(w^*)$, a fraction $1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)$ will continue buying from him. Therefore, the deviating retailer's profit in a uniform pricing equilibrium equals:

$$\pi_r(\widetilde{p}, p^*) = \frac{1}{N}\left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right) D(\widetilde{p})(\widetilde{p} - w^*).$$

Maximizing retail profit and using the equilibrium condition $\widetilde{p}(w^*) = p^*(w^*)$, yields

$$-g(0)D^2(p^*)(p^* - w^*) + D^{'}(p^*)(p^* - w^*) + D(p^*) \leq 0. \tag{1}$$

Note that for a given w^* the equilibrium retail price is independent of the number of active retailers and that $p^* \leq p^M(w^*)$. Note also that, in principle, *from the perspective of retailers* the first-order condition can be satisfied with a weak inequality as retailers will never have an incentive to lower their price as long as $p^* \leq p^M(w^*)$: given that consumers search and do not observe these lower prices until at the retailer in question, retailers do not attract more consumers by lowering their prices. Note that this implies there is a continuum of pure-strategy equilibria at the retail level including the retail monopoly price. In the next Section, we argue that in *the full vertical model*, taking the incentive of the manufacturer into account, it can never be the case that (1) holds with strict inequality and this is what we focus on now.

To illustrate the impact of wholesale price discrimination on the retail market, consider the situation where a manufacturer is expected to set a wholesale price of w_L^* to $N-1$ retailers and w_H^* to 1 retailer,[10] but consumers do not know which retailer faces the higher wholesale price. The low and high cost retailers, on their part, are expected to react by setting p_L^* and p_H^*, respectively. The first effect of wholesale price discrimination on consumer search is that the low search cost consumers who happen to encounter the high cost retailer setting p_H^* will continue to search for lower retail prices. In particular, defining $\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$, all consumers who happen to observe p_H^* at their first search and have a search cost $s < \widehat{s}$ continue to search as updating beliefs using Bayes' rule implies that consumers believe all other retailers set p_L^*.[11]

To understand how retailers will react to wholesale price discrimination, we have to be more specific here about consumer beliefs and go beyond passive beliefs: after the observation of an out-of-equilibrium price, consumers should also have beliefs about whether a high or a low cost retailer has deviated. Retail equilibrium requires that at prices p in the neighbourhood of p_H^* consumers believe it is a high-cost retailer that has deviated. The reason is as follows. If the high-cost retailer sets the equilibrium price p_H^* his profit equals

$$\pi_r^{H*} = \frac{1}{N}\left(1 - G\left(\widehat{s}\right)\right) D(p_H^*)(p_H^* - w_H^*).$$

If consumers attribute the deviation price to a low cost retailer, then after observing a price $p_H \neq p_H^*$ they become more pessimistic about finding lower prices on their next search than after observing p_H^*. In particular, they would believe there is a probability $\frac{1}{N-1}$ that they encounter a high-cost retailer on their next search, so that it takes them an expected search cost of $\frac{N-2}{N-1}s + \frac{1}{N-1}2s = \frac{N}{N-1}s$ to find a lower price. Thus, these first time consumers encountering a price $p_H > p_H^*$ will continue to search if their search cost is $s < \frac{N-1}{N}\int_{p_L^*}^{p_H} D(p)dp$. More consumers would then decide not to continue searching if they observe such a deviation price than after observing p_H^*, but this would make it profitable

[10] In the main part of the paper this is also the setting we consider and in the online Appendix we discuss why.

[11] If there would be $m^* < N-1$ retailers getting a low wholesale price, then the critical search cost value $\widehat{s}$ would be defined as:

$$\left(\frac{m^*}{N-1} + \frac{N-m^*-1}{N-1}\frac{m^*}{N-2} + .. + \frac{N-m^*-1}{N-1}\frac{N-m^*-2}{N-2} \cdot .... \cdot 1\right)\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$$

as there is a chance that consumers will not immediately encounter p_L^* on their next search.

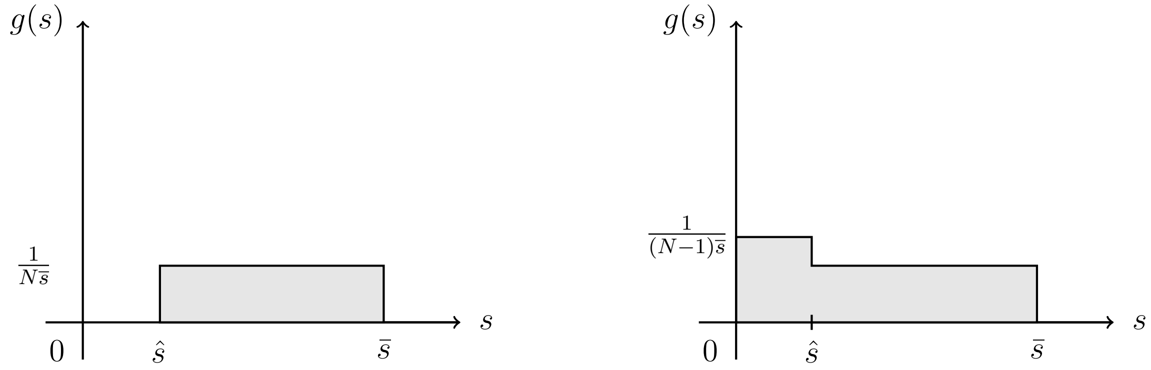


Figure 2: **Left:** Search cost compositions of demand for a high cost retailer. **Right:** Search cost compositions of demand for a low cost retailer; where $s \sim U[0, \overline{s}]$.

for a high cost retailer to deviate. Thus, specifying that after observing a price p_H in the neighbourhood of p_H^* consumers blame a high cost retailer for the deviation, they will continue to search if their search cost is such that

$$s < \widehat{s} + \int_{p_H^*}^{\overline{p}} D(p)dp - \int_{p_H}^{\overline{p}} D(p)dp = \int_{p_L^*}^{p_H} D(p)dp.$$

The left panel of Figure 2 illustrates the search cost composition of demand for the high cost retailer, when search costs are uniformly distributed on the interval $[0, \overline{s}]$.

Therefore, the profit of a retailer who has a wholesale price w_H^* and sets a price p_H in the neighbourhood of p_H^* will be:

$$\pi_r^H(p_H, p_L^*; w_H^*) = \frac{1}{N}\left(1 - G\left(\int_{p_L^*}^{p_H} D(p)dp\right)\right) D(p_H)(p_H - w_H^*). \tag{2}$$

Consider now a low cost retailer contemplating a deviation to a price p_L in the neighbourhood of p_L^*. As in any costly sequential search model, downward deviations are not optimal as they do not attract additional demand. Consider then an upward deviation. Here, we are free to specify which retailer consumers blame for such a deviation. The equilibrium price level p_L^* depends, of course, on how we specify these beliefs. The higher the fraction of consumers blaming upward deviations on the high cost retailer, the more competitive the retail market will become as more consumers will continue searching after observing an upward deviation from p_L^*. In the full model considered in the next Sections, a more competitive retail market implies higher profits for the manufacturer. As we do not want our results to be driven by arbitrary out-of-equilibrium beliefs that favour retail competition, we assume that consumers attribute deviations to a low cost retailer if the deviation price p_L is in the neighbourhood of p_L^*. This also implies that beliefs are continuous in a neighbourhood of both equilibrium prices.[12] At the end of this Section we will determine the retail price p_L^* under alternative beliefs.

[12]Consumers observing the equilibrium price p_L^* believe that if they continue to search, there is a probability of $\frac{1}{N-1}$ they will observe a price p_H^* on their next search. Consumers observing the equilibrium price p_H^* believe that there is zero probability that they will observe a price p_H^* on their next search.

Given these beliefs, there are two important differences with the case of uniform pricing in evaluating the profitability of an upward deviation by the low cost retailer. First, consumers are less inclined to continue searching compared to the uniform pricing case as now there is a positive probability that they will encounter an even higher retail price on their next search. We call this the *anti-competitive effect* of wholesale price discrimination. As low search cost consumers will continue to search until they find the lowest expected price p_L^* in the market, the benefit of search equals $\int_{p_L^*}^{p_L} D(p)dp$, whereas the expected cost of search equals $\frac{N-2}{N-1}s + \frac{1}{N-1}2s = \frac{N}{N-1}s$. Thus, these first time consumers encountering a price p_L will continue to search if their search cost is $s < \frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp$.

For a low cost retailer contemplating a deviation to a price $p_L > p_L^*$ there is, however, an important other effect of wholesale price discrimination on consumer search. Due to the fact that low search cost consumers continue to search if they observe p_H^* on their first search, low cost retailers will serve a disproportionately larger share of low search cost consumers. Therefore, they are losing relatively more consumers if they deviate and increase their prices. We call this the *screening effect* of wholesale price discrimination and illustrate it in the right panel of Figure 2.

Combining these two effects, when deviating to a price p_L, with $p_L^* < p_L < p_H^*$, a low cost retailer's profit function will be:

$$\pi_r^L(p_L; p_L^*, p_H^*, w_L^*) = \frac{1}{N}\left[1 - G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{G\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{(N-1)}\right] D(p_L)(p_L - w_L^*). \tag{3}$$

Thus, there are two important differences in this profit function relative to the uniform pricing case. First, the term $\frac{N-1}{N}$ in the first $G(s)$ function reflects the *anti-competitive effect* described above. The last term in the square brackets reflects the *screening effect* of low cost retailers having a disproportionately large share of low search cost consumers.

The different effects of wholesale price discrimination on consumer search have important implications for competition in the retail market as can be seen from taking the first-order conditions of the profit functions of the different retailers. Taking the first-order condition of (2) with respect to p_H and substituting $p_H = p_H^*$ yields

$$-\frac{g(\widehat{s})D^2(p_H^*)(p_H^* - w_H^*)}{1 - G(\widehat{s})} + \left[D'(p_H^*)(p_H^* - w_H^*) + D(p_H^*)\right] = 0. \tag{4}$$

First, note that the FOC condition has to hold with equality as a high-cost retailer may also have an incentive to lower price to prevent more consumers from continuing to search. Second, comparing this FOC condition with that in (1) reveals that *ceteris paribus* the only difference is that the first term is multiplied by the hazard rate $\frac{g(\widehat{s})}{1-G(\widehat{s})}$ instead of by $g(0)$. As this first term is negative, this implies that high cost retailers will have lower margins if, and only if, $\frac{g(\widehat{s})}{1-G(\widehat{s})} > g(0)$. This is certainly the case if the search cost

Thus, also on the-equilibrium path beliefs about retail prices on the next search depend on which retail price is observed.

distribution has an increasing hazard rate (which is true for many distributions). This is one of the important effects of wholesale price discrimination discussed in the Introduction: as (some) retailers have lower retail prices, it is more attractive for consumers to continue searching if they have visited a high cost retailer, which imposes a more severe competitive constraint on these retailers. High cost retailers have fewer buying customers (represented by $1-G(\widehat{s})$) and an upward deviation from the equilibrium price will cause $g(\widehat{s})$ consumers to leave relative to $g(0)$ in the uniform pricing equilibrium. In relative terms, the impact of consumers leaving is greater on the high cost retailer.

Taking the first-order condition of (3) with respect to p_L and evaluating it at the equilibrium value yields:

$$-\frac{\left(\frac{(N-1)^2}{N}g(0)+g(\widehat{s})\right)D^2(p_L^*)(p_L^*-w_L^*)}{(N-1)+G(\widehat{s})}+\left[D'(p_L^*)(p_L^*-w_L^*)+D(p_L^*)\right]\leq 0. \quad (5)$$

Comparing this FOC with that in (1) reveals that *ceteris paribus* the only difference is that the first term is multiplied by $\frac{\frac{(N-1)^2}{N}g(0)+g(\widehat{s})}{(N-1)+G(\widehat{s})}$ instead of $g(0)$. The easiest way to compare these two terms is for the uniform distribution where $g(s)$ is constant. In that case, the term in (5) is larger than $g(0)$ if, and only if, $G(\widehat{s})\leq 1/N$. Especially, when N is small, this term creates an important difference and illustrates an important effect of wholesale price discrimination as discussed in the Introduction: even though low search cost consumers may be less inclined to continue to search (as they may not directly find another low cost retailer), the fact that low cost retailers are more frequently visited by low search cost consumers outweighs this effect.

If we would have specified out-of-equilibrium beliefs differently, so that consumers would always blame a high cost retailer for having deviated, then a low cost retailer's deviation profit function would have been:

$$\pi_r^L(p_L;p_L^*,p_H^*,w_L^*)=\frac{1}{N}\left[1-G\left(\int_{p_L^*}^{p_L}D(p)dp\right)+\frac{G\left(\int_{p_L}^{p_H^*}D(p)dp\right)}{(N-1)}\right]D(p_L)(p_L-w_L^*).$$

Comparing this to the deviation profit under uniform pricing, one easily sees that the only important difference is the third term in the square brackets. This is the *screening effect* of wholesale price discrimination which is pro-competitive. Thus, under this specification of out-of-equilibrium beliefs, low-cost retail margins would always be smaller under wholesale price discrimination compared to uniform pricing.

The above discussion shows that *ceteris paribus* retail margins are generally lower under wholesale price discrimination. *Ceteris paribus* here mainly is a clause relating to wholesale prices: the first-order conditions are similar to the first-order retail condition under uniform pricing if evaluated at the *same* wholesale price. The important question then is how these changes in the first-order retail price conditions impact the manufacturer's incentives to set wholesale prices. It should be clear that it is not optimal for the manufacturer to induce an equilibrium where $\widehat{s} \geq \overline{s}$. If that would be an equilibrium, retailers receiving a high wholesale offer, reacting with a retail price p_H^*, would be effectively foreclosed from the market, putting the remaining retailers in the same position as under uniform pricing. Thus, in the remaining we consider $0 < \widehat{s} \leq \overline{s}$.

4 Commitment

In this section, we compare uniform pricing to wholesale price discrimination in case the manufacturer is able to commit to wholesale prices. We interpret commitment here as the case where all retailers and consumers observe the contractual arrangements set by the manufacturer.[13] This allows us to focus on the impact of wholesale price discrimination on the retail market without having to consider the different beliefs retailers and consumers may have about wholesale contracts. The only belief that is relevant is the belief of consumers about retail prices.

One way to think of this commitment is that manufacturers have long-term contracts with retailers, that consumers know about this and that the latter repeatedly buy. Consumers understand that the retail price they observe is the result of a decision by the manufacturer and the retailer. In this case, consumers may learn about which retail prices to expect (and thus indirectly infer the underlying wholesale prices) on the basis of their own previous shopping experiences or through word-of-mouth communication via their friends. For markets where this form of commitment is *not* possible, the commitment case can be seen as a convenient theoretical benchmark.

In this section, commitment to uniform pricing refers to the case that all retailers and consumers know (or believe) that the manufacturer always sets the same wholesale price to all retailers. Commitment to wholesale price discrimination refers to the case where consumers and retailers know that some retailers have obtained one wholesale price whereas others have obtained a different wholesale price. As it is essential to our theory that consumers cannot direct their first search to some particular retailer (as they are considered to be symmetric), one can re-interpret the wholesale price discrimination case as one in which the manufacturer gives all retailers identical wholesale contracts in which it is stipulated that a fixed number of retailers gets one wholesale price and the remaining retailers get the other wholesale price and that who gets which wholesale price is randomly determined. In terms of long-term contracts, the contract to all retailers may then stipulate that in every period a fixed number of retailers can buy at w_H, while the others get a discount and effectively pay w_L, and that at the beginning of every period, the retailers that get the discount are randomly selected. In the next Section, where

[13] In a study of first-mover advantage, Bagwell [1995] has shown that a player's ability to commit is equivalent to the observability of his actions. In our world, with a manufacturer, multiple retailers and many consumers, the issue of commitment is more subtle as the manufacturer may commit to an individual retailer, or to retailers in general, without committing to consumers.

consumers do not know the wholesale contracts, the manufacturer may price discriminate and choose deterministic contracts as explained there.

It is clear that at least two retailers should get the lowest price. The reason is that if one retailer knows it is getting the lowest price, then it does not face any competition from other retailers up to the second lowest equilibrium retail price in the market. Therefore, this retailer would then set a retail price (almost) equal to the second lowest equilibrium retail price in the market, giving the manufacturer an incentive to increase the lowest wholesale price. Thus, to keep a competitive constraint on the retailers receiving the lowest wholesale price, there should be at least two retailers being offered w_L. In the case of $N = 3$, wholesale price discrimination implies that two retailers buy at w_L and one buys at w_H. For $N > 3$, the question of how many retailers should get the lowest wholesale price is non-trivial. We focus on showing that wholesale price discrimination in its more simple form where the manufacturer chooses to set a low wholesale price w_L to $N-1$ retailers and another high wholesale price w_H to 1 retailer increases manufacturer profits.[14] Retailers will react to these prices by setting (possibly) different retail prices. We denote by $p_L^*(w_L, w_H)$ and $p_H^*(w_L, w_H)$ the retail price a low, respectively high, cost retailer sets when wholesale prices are w_L and w_H and $N-1$ retailers receive the lowest offer. Even though a retailer may only be directly interested in his own wholesale price, the other retail price (and thus the wholesale price) is of relevance as it determines consumers' search behaviour.

We summarize by providing the equilibrium definition with wholesale price discrimination used in this section. For a discussion on consumers' out-of-equilibrium beliefs we refer the reader to the previous Section.[15] As the equilibrium definition of the benchmark with uniform pricing is a special case, we skip that formal definition.

Definition 1 *If the manufacturer can commit, an equilibrium with wholesale price discrimination is defined in two parts. First, for every w_L and w_H we define retail strategies $p_L^*(w_L, w_H)$ and $p_H^*(w_L, w_H)$ and an optimal sequential search strategy for all consumers such that (i) retailers maximize their retail profits given consumers' optimal search strategy and (ii) consumers' sequential search strategy is optimal given their beliefs. Consumer beliefs are updated using Bayes' rule whenever possible. If consumers observe a retail price p in the neighbourhood of $p_H^*(w_L, w_H)$ they believe that a high cost retailer is responsible for setting this price. Second, given these behaviours, the manufacturer chooses w_L and w_H to maximize her profits.*

As discussed in the previous Section, not to have our results driven by arbitrary out-of-equilibrium beliefs that favour retail competition, we will assume that consumers attribute

[14]In an online Appendix we discuss how the analysis would be affected if more retailers receive the highest wholesale price.

[15]It is the out-of-equilibrium beliefs regarding local deviations that determine the equilibrium levels. We specify other out-of-equilibrium beliefs such that the equilibrium definitions are fulfilled and in the proofs we show that we can do so.

deviations to a low cost retailer if the deviation price p is in the neighbourhood of p_L^*. Out-of-equilibrium beliefs after deviations to other prices are specified such that they are continuous and such that these deviations are not profitable. This allows us to use in this section the derivations of the previous section to describe retail and consumer behaviour for given wholesale prices. Essentially, we add a stage where the manufacturer chooses wholesale price(s) to maximize profits taking the reactions to her prices as given.

Uniform pricing

Under commitment to uniform pricing, the manufacturer chooses a wholesale price w that maximizes her profit $\Pi^M = wD(p(w))$, where $p(w)$ is implicitly defined by (1) holding with equality. The reason that, in *the vertical model*, (1) has to hold with equality is that otherwise it would be profitable for the manufacturer to marginally increase her wholesale price as retailers would not adjust their retail prices and therefore the manufacturer's demand would not be affected, increasing the manufacturer's profits.

Thus, with uniform pricing and commitment the wholesale price w is set such that[16]

$$\frac{\delta \Pi^M}{\delta w} = wD'(p(w))\frac{\delta p^*}{\delta w} + D(p(w)) = 0. \tag{6}$$

To determine the optimal wholesale price we still have to evaluate $\frac{\partial p^*}{\partial w}$. Taking the total differential of (1) it follows that

$$\frac{\partial p^*}{\partial w} = \frac{D'(p^*) - g(0)D^2(p^*)}{-2g(0)D(p^*)D'(p^*)(p^* - w) - g(0)D^2(p^*) + (D''(p^*)(p^* - w) + 2D'(p^*))}.$$

For search cost distributions that are not concentrated around 0, it is difficult to characterize the equilibrium beyond the above expressions or to obtain comparative statics results. If the search cost distribution is concentrated close to 0, however, we can provide the following characterization. For any concentrated search cost distribution, $g(0)$ is very large. From (1) it can be seen that as $g(0) \to \infty$, $p^*(w^*) \to w^*$. This is quite intuitive: when all consumers have arbitrarily small search cost, retailers do not have any market power and their retail margins should become arbitrarily small as well. If $g(0) \to \infty$ and $p^* \to w^*$, the expression for $\frac{\partial p^*}{\partial w}$ reduces to 1 so that the wholesale price is equal to that of an integrated monopolist.

The following Proposition summarizes this result and analyses the limiting behaviour of the wholesale and retail price in a neighbourhood of $\overline{s} = 0$.

Proposition 2 *Suppose the manufacturer commits to uniform pricing. For $\overline{s}$ close enough to 0, an equilibrium exist and is unique. If $\overline{s} \to 0$, then the uniform retail and wholesale prices converge to $p^* = w^*$, where w^* solves $w^*D'(w^*) + D(w^*) = 0$. Moreover, $\frac{dp^*}{d\overline{s}} = 0$ and $\frac{dw^*}{d\overline{s}} = -\frac{1}{D(p^*)}$ so that $\frac{d\Pi^M}{d\overline{s}} = -1$.*

[16] Under commitment to uniform pricing, the manufacturer has to set the same wholesale price to all retailers, so if she deviates from the equilibrium level, she has to do so to all retailers.

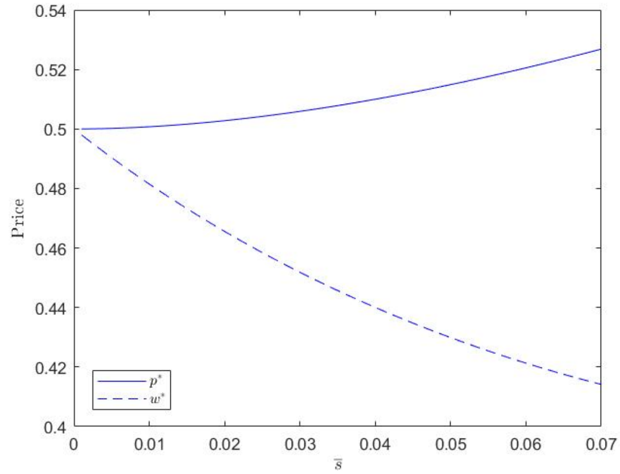


Figure 3: Uniform retail and wholesale prices under commitment for different values of $\overline{s}$

For search cost distributions that are not concentrated around 0, the general expressions provided above allow us to solve the model numerically, for different demand functions and search cost distributions. To compare numerical results across different environments, we focus on the case of linear demand $D(p) = 1 - p$ and a uniform search cost distribution, where $g(s) = 1/\overline{s}$. Figure 3 clearly shows that the retail price increases, while the wholesale price decreases in reaction to an increasing support of the search cost distribution: when retailers have more market power because of the increasing importance of search costs, the manufacturer prevents a larger decrease in demand by lowering the wholesale price. As a result, retail profits are increasing, the manufacturer profit is decreasing and consumers are worse off.

Wholesale Price Discrimination

Under commitment and wholesale price discrimination, the manufacturer will choose two different wholesale prices, w_L and w_H, to directly maximize profit:

$$\Pi^M(w_L, w_H) = \frac{1}{N}\left[1 - G(\widehat{s})\right] w_H D(p_H^*(w_L, w_H)) + \frac{N - 1 + G(\widehat{s})}{N} w_L D(p_L^*(w_L, w_H)).$$

Under commitment, the manufacturer takes into account how retail prices change in reaction to changes in w_L and w_H. To derive these reactions, we have to consider that the retail first-order conditions (4) and (5) stipulate that a retailer's reaction to its own wholesale price depends on the other retail price they expect through its impact on $\widehat{s}$. That is, (5) describes a relationship where p_L depends on w_L and p_H and (4) describes a relationship where p_H depends on w_H and p_L. Thus, for every fixed pair of wholesale prices (w_L, w_H) we can solve for the retail reactions by simultaneously solving (4) and (5). In this way, the retail equilibrium reactions are given by $p_L^*(w_L, w_H)$ and $p_H^*(w_L, w_H)$, i.e., both retail prices depend directly on the corresponding wholesale price, but also indirectly on the other wholesale price, through its influence on the other retail price.

Using these reactions, we solve the profit maximization problem:

Proposition 3 *Suppose the manufacturer commits to wholesale price discrimination. For $\overline{s}$ close enough to 0, an equilibrium with $w_L^* < w_H^*$ exists and is unique. If $\overline{s} \to 0$, the wholesale prices w_L^* and w_H^* and retail prices p_L^* and p_H^* converge to w^* and p^*, with $p^* = w^*$ solving $w^* D^{'}(w^*) + D(w^*) = 0$. The retail margins are given by:*

$$\frac{d\left(p_L^* - w_L^*\right)}{d\overline{s}} = \left(\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)} + \frac{N\left(2N-1\right)}{\left(2N^2-N+2\right)}\right)\frac{1}{D(p_L^*)},$$
$$\frac{d\left(p_H^* - w_H^*\right)}{d\overline{s}} = \left(-\frac{1}{2\left(N^2+1\right)} + \frac{2N^2-N+1}{2N^2-N+2}\right)\frac{1}{D(p_L^*)},$$

while $\frac{d\Pi^M}{d\overline{s}} > -1$.

Thus, we can unambiguously state that the manufacturer makes more profit by price discriminating. If the manufacturer can commit, she will not commit to uniform pricing as this yields lower profits than committing to price discrimination. Indeed, as $\frac{d\left(p_i^* - w_i^*\right)}{d\overline{s}} < \frac{1}{D(p_L^*)} = \frac{d(p^*-w^*)}{d\overline{s}}, i = L, H$, it is clear that both retailers make lower margins under wholesale price discrimination than under uniform pricing.

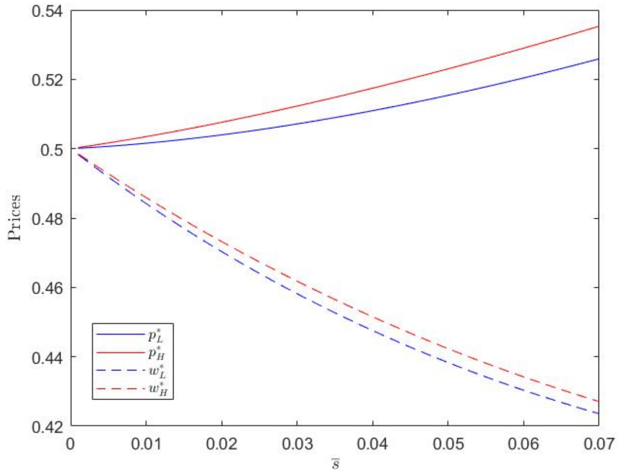
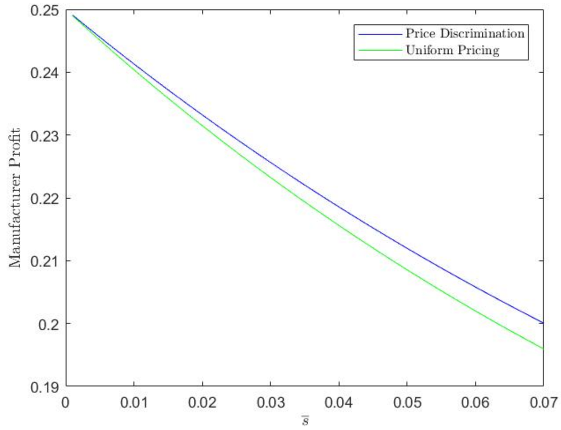


Figure 4: Prices under wholesale price discrimination for different values of $\overline{s}$

The above result shows that the manufacturer is better off, while retailers are worse off, because of wholesale price discrimination. What is not clear, however, from the above first-order approximations is whether or not consumers are better off. This depends on the retail prices and not on the retail margins. As the first-order approximations leave the price levels undetermined and a second-order approximation is extremely tedious, we provide, in Figure 4(Right), the following numerical analysis for linear demand and a uniform search cost distribution clearly showing that consumers are also worse off. [17]

[17] Note that the horizontal axis reports values of $\overline{s}$ between 0 and 0.07. It may seem that a search cost of 0.07 is still quite small. Note, however, that for linear demand $D(p) = 1 - p$ expected consumer surplus is also quite small and that this search cost is already more than half of the expected surplus.



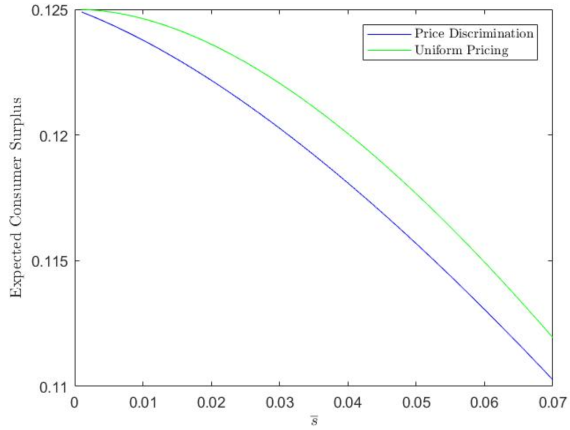


Figure 5: **Left:** Manufacturer's profit under commitment for different values of $\overline{s}$. **Right:** Expected Consumer Surplus under commitment for different values of $\overline{s}$.

5 No commitment

Without commitment, the manufacturer may secretly deviate from the prices retailers and consumers expect her to charge. A retailer only observes her own wholesale price and does not observe the wholesale arrangements of the other retailers. Consumers only observe the retail price they encounter when searching and do not know the wholesale arrangements. Thus, in this section, we should not only consider consumers', but also retailers' out-of-equilibrium beliefs. Moreover, in this section, we can have pure strategy equilibria that are consistent with consumers not knowing which retailer has the high wholesale price (where in the previous Section the manufacturer had to write into the contract that some randomly chosen retailers get a discount).

The literature on unobservable contracts (where consumer search is not considered), has shown that a manufacturer may be subject to opportunism when contracting secretly with downstream retailers and that equilibrium behaviour depends on the type of beliefs retailers hold. We follow the approach used by the seminal papers in this literature (see, Hart and Tirole [1990], O'Brien and Shaffer [1992] and McAfee and Schwartz [1994]) and assume that retailers hold passive beliefs. We define a Perfect Bayesian Equilibrium (PBE) with passive out-of-equilibrium beliefs and wholesale price discrimination as follows.

Definition 4 *If the manufacturer cannot commit, an equilibrium with wholesale price discrimination is defined by a tuple* $((w_L^*, w_H^*), p^*(w))$*, with* $w_L^* < w_H^*$*, and an optimal sequential search strategy for all consumers such that (i) the manufacturer maximizes profits given* $p^*(w)$ *and consumers' optimal search strategy, (ii) retailers maximize their retail profits given the wholesale price they observe, their beliefs about the wholesale prices received by other retailers and consumers' optimal search strategy and (iii) consumers' sequential search strategy is optimal given* $((w_L^*, w_H^*), p^*(w))$ *and their beliefs about retail prices not yet observed. Beliefs are updated using Bayes' rule whenever possible. Off-the-equilibrium path beliefs are passive and satisfy at least the following restrictions:*

- *If consumers observe a retail price p in the neighbourhood of $p^*(w_H^*)$ they believe that a high cost retailer is responsible for setting this price;*
- *A retailer observing a wholesale price w in the neighbourhood of w_H^* believes that all other competitors receive a wholesale price of w_L^*.*

Similar to the previous two Sections, and not to have the profitability of wholesale price discrimination be driven by beliefs, the analysis below assumes similar belief formation around w_L^* and p_L^*. This is not part of the formal definition, however, as we could have chosen to use different beliefs around w_L^* and p_L^*. As uniform pricing is a special case, we skip the formal definition.

Uniform pricing

To determine the wholesale equilibrium price under uniform pricing without commitment, we should consider that it is not optimal for the manufacturer to deviate to one retailer and offer him a wholesale price w (keeping the other retailers at w^*). If the manufacturer would deviate in this way and the retailer would react to w by choosing $\widetilde{p}$ (to be determined later), her profits would be:

$$\pi(w^*, w) = w^* D(p^*(w^*)) + \frac{1}{N}\left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right)\left(wD(\widetilde{p}(w)) - w^* D(p^*(w^*))\right).$$

This expression is easily understood. Of the consumers who encounter a price of $\widetilde{p}(w)$ at their first search (which is a fraction $1/N$ of them) a fraction $G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)$ continues to search for the equilibrium retail price as their search cost is low enough, while the consumers with a search cost larger than $\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp$ will buy at the deviation price $\widetilde{p}(w)$. All other consumers buy at the equilibrium price $p^*(w^*)$. A uniform pricing equilibrium requires that the first-order condition evaluated at $w = w^*$ is non-positive, i.e.,

$$g(0)D(\widetilde{p})\frac{\partial \widetilde{p}}{\partial w}\left(w^* D(p^*(w^*)) - wD(\widetilde{p}(w))\right) + \left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right)\left(wD'(\widetilde{p})\frac{\partial \widetilde{p}}{\partial w} + D(\widetilde{p})\right)$$

$$= \left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right)\left(w^* D'(p^*)\frac{\partial \widetilde{p}(w^*)}{\partial w} + D(p^*)\right) \leq 0,$$

which reduces to

$$w^* D'(p^*(w^*))\frac{\partial \widetilde{p}(w^*)}{\partial w} + D(p^*(w^*)) \leq 0.^{18} \tag{7}$$

[18] In a different context, Rey and Vergé [2004] have shown that an equilibrium where retailers hold passive beliefs may not exist. It is clear that condition 7 guarantees that the manufacturer does not have an incentive to deviate to multiple or even all retailers (provided that retailers hold passive beliefs). Therefore, in contrast to Rey and Vergé [2004], assuming that retailers hold passive beliefs does not lead to non-existence results in our setting.

Similar to the retailer's behaviour considered in Section 3, the manufacturer does not have an incentive to lower his wholesale price as long as $p^* < \min(p^M(w^*), p^M(w^M))$ as retailers will not follow suit and keep their price at the equilibrium level if this condition is satisfied. In this case, the only requirement we have to impose is that the manufacturer does not want to increase his wholesale price and this is what (7) requires. On the other hand, nothing we have said so far precludes the possibility that the solutions to (1) and (7) result in such a high wholesale (and retail) price that $w^*D(p^*(w^*)) < w^M D(p^M(w^M))$. In this case, it would be optimal, however, for the manufacturer to deviate to all retailers by setting w^M and they will respond by setting $p^M(w^M)$. Thus, another condition that an equilibrium needs to fulfil is that the manufacturer's equilibrium profit satisfies $w^*D(p^*(w^*)) \geq w^M D(p^M(w^M))$.

To finalize the description of an equilibrium, we still have to evaluate how $\widetilde{p}$ depends on the deviation wholesale price w. For this we need to determine the best response function of retailers to non-equilibrium wholesale prices, taking into account that now consumers do not observe the manufacturer deviation and blame the individual retailer for any deviation from the equilibrium price. Thus, without commitment, retailers will react differently to a change in w relative to the commitment case. Given the retailers' profit function $\pi_r(\widetilde{p}, p^*) = \frac{1}{N}\left(1 - G\left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp\right)\right) D(\widetilde{p})(\widetilde{p} - w)$, an individual retailer will react to upward deviations from w^* by setting $\widetilde{p}$ such that

$$-g\left(\int_{p^*}^{\widetilde{p}} D(p)dp\right) D^2(\widetilde{p})(\widetilde{p} - w) + \left(1 - G\left(\int_{p^*}^{\widetilde{p}} D(p)dp\right)\right)\left(D^{'}(\widetilde{p})(\widetilde{p} - w) + D(\widetilde{p})\right) = 0. \tag{8}$$

Thus, the retailer's best response to any w depends on w itself as well as on the equilibrium price p^* that is expected by consumers. Under no commitment, the retailer should not only consider the wholesale price itself, but also how consumers who do not observe the wholesale price react (and this depends on the retail prices they expect). In the proof of the next Proposition we show that evaluated at the equilibrium values we obtain:

$$\frac{\partial \widetilde{p}(w^*)}{\partial w} = \frac{D'(p^*) - g(0)D^2(p^*)}{-g^{'}(0)D^3(p^*)(p^* - w) - 3g(0)D(p^*)D^{'}(p^*)(p^* - w) - 2g(0)D^2(p^*) + 2D'(p^*) + 2D^{''}(p^*)(p^* - w)}. \tag{9}$$

We then have the following result.

Proposition 5 *Suppose the manufacturer cannot commit to uniform pricing. A uniform pricing equilibrium has to satisfy (1), (7), where* $\frac{\partial \widetilde{p}(w^*)}{\partial w}$ *is given by (9) and* $w^*D(p^*(w^*)) \geq w^M D(p^M(w^M))$.

If an equilibrium exists, there can be multiple equilibria due to the fact that the first-order condition of the manufacturer only needs to hold with inequality. We focus on the equilibrium where the manufacturer makes most profits. This is the equilibrium where (7) holds with equality. Equilibria can be indexed by the wholesale price that retailers and consumers expect the manufacturer to choose. As the manufacturer is a monopolist, we believe it is natural to think that retailers and consumers expect that the manufacturer chooses the equilibrium wholesale price that maximizes her profits, which is the lowest of all equilibrium wholesale prices and is thus also in the interest of consumers.

As in the commitment case, if $g(0) \to \infty$ we have that $p^*(w^*) \to w^*$. What is perhaps more surprising is that when $g(0) \to \infty$ and $p^* \to w^*$ we can solve (7) for w^*. From (9) it is easy to see that if $g(0) \to \infty$ the expression for $\frac{\partial \tilde{p}(w^*)}{\partial w}$ reduces to $\frac{1}{2}$ so that the wholesale price is significantly larger than that of an integrated monopolist. It is quite intuitive that retailers will not react as strongly to wholesale prices as under commitment. Under uniform pricing in the previous section, the manufacturer sets the same wholesale price to all retailers so if she sets a non-equilibrium price she does so to all retailers and consumers know this. Without commitment consumers believe that all other retailers set the equilibrium price and more consumers will continue to search if a retailer does not choose the price consumers expected. The next Proposition states the result.

Proposition 6 *Consider $\overline{s}$ small enough. If $\partial p^M(w^M)/\partial w < 1$,*[19] *a uniform pricing equilibrium exists and any uniform pricing equilibrium converges to $p^* = w^*$, where w^* solves $\frac{1}{2}w^* D'(w^*) + D(w^*) \leq 0$. Moreover, $\frac{dp^*}{d\overline{s}} = -\frac{x}{D(p^*)} < 0$ and $\frac{dw^*}{d\overline{s}} = -\frac{1+x}{D(p^*)} < 0$, where $x = \frac{2D'(p^*)}{w^* D''(p^*)+3D'(p^*)}$.*

In the context of a Stahl (1989) type model, where a fraction λ of consumers (the shoppers) has zero search cost and the remaining consumers have a search cost $s > 0$, Janssen and Shelegia [2015] show that if the search cost s is small an equilibrium exists if, and only if, λ is large enough.[20] The first part of the above Proposition says that if the search cost is small equilibrium existence is generally not an issue in our model where consumers have truly heterogeneous search cost and $g(s) > 0$ for all $s \geq 0$. Thus, our result shows that the equilibrium in-existence result in Janssen and Shelegia [2015] is due to the discreteness of the search cost distribution.[21]

The second part of the Proposition establishes that the manufacturer sets a much higher price than an integrated monopolist. This result is akin to Theorem 2 of Janssen and Shelegia [2015] where they show that as $s \to 0$, wholesale and retail prices converge to a price w^* that solves $\lambda w^* D'(w^*) + D(w^*) = 0$. The reason why equilibrium prices are much higher than the price an integrated monopolist would set (despite the retail

[19]The condition that the cost pass-through evaluated at the double marginalization price is smaller than 1 guarantees that $w^* D(p^*(w^*)) \geq w^M D(p^M(w^M))$. Weyl and Fabinger [2013] derive the cost pass-through in terms of primitives of the demand curve. We have not been able to find demand curves that satisfy that $D(p) = 0$ for $p > \overline{p}$ for which $w^* D(p^*(w^*)) < w^M D(p^M(w^M))$ in a neighbourhood of $\overline{s} = 0$.

[20]For linear demand, the critical value λ^* is approximately 0.47.

[21]From Janssen and Shelegia [2015], we know that we cannot guarantee existence for any search cost distribution that is non-concentrated ($\overline{s}$ is small), even if demand is linear. In an online Appendix we show, by means of a numerical analysis that at least for linear demand and a uniform search cost distribution, that the existence of equilibrium is not an issue even when $\overline{s}$ is larger.

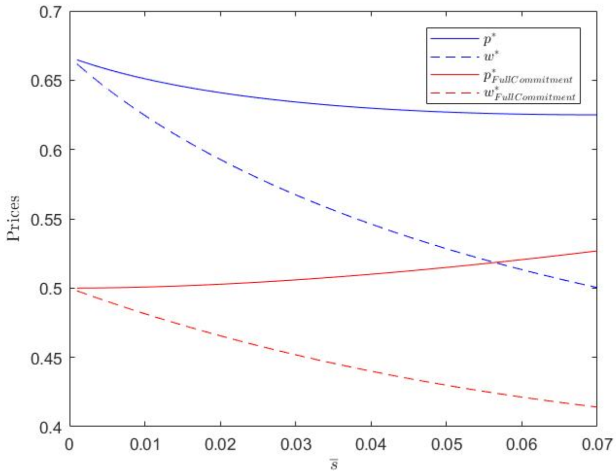


Figure 6: Uniform retail and wholesale prices for different values of $\overline{s}$

margins being close to 0) is that the manufacturer may deviate from the equilibrium price without consumers noticing it. This makes the manufacturer's demand much less elastic to her own price changes than the demand of an integrated monopolist. Theorem 2 of Janssen and Shelegia [2015] is obtained for duopoly retail markets and the Stahl (1989) specification of search costs. The above result shows that the intuition is much more general and holds for any search cost distribution and for any number of retailers. Also, as in Janssen and Shelegia [2015], an equilibrium only exists if λ is large enough, their limit prices tend to be (much) smaller than in our model.

In terms of comparative statics, Proposition 6 shows that in a neighbourhood of $\overline{s} = 0$ both the wholesale and retail price are decreasing in $\overline{s}$. This implies that consumers are better off if search costs are not vanishing. Janssen and Shelegia [2015] have a similar result, but only for the case of linear demand. This result indicates that price comparison websites that effectively reduce search costs and are believed to help consumers in getting better deals may in the end lead to higher prices.

For linear demand $D(p) = 1 - p$, the Proposition implies that in the limit when $\overline{s} \to 0$, $w^* \to 2/3$ and expected consumer surplus converges to $\frac{1}{18}$.[22] Using Proposition 6, we have that $\frac{dp^*}{d\overline{s}} = -2$, $\frac{dw^*}{d\overline{s}} = -5$ and $\frac{dESC}{d\overline{s}} = -(1-p^*)\frac{dp^*}{d\overline{s}} = \frac{2}{3}$. Figure 6 shows how the equilibrium retail and wholesale prices change for different values of $\overline{s}$. For small values of $\overline{s}$ the figure also confirms that both p^* and w^* are decreasing in $\overline{s}$. The figure also depicts the retail and wholesale prices under commitment and confirms that without commitment prices are much higher than under commitment and that retail prices behave differently in these two cases: under commitment, uniform retail prices are increasing in the upper bound of the search cost distribution, while they are decreasing without commitment.

[22] Other equilibria are such that $w^* \geq 2/3$, while the condition that deviation to the double marginalization solution is not optimal results in the condition $w^*(1-w^*) \geq 1/8$, or $w^* \leq \frac{2+\sqrt{2}}{4}$.

Wholesale Price Discrimination

We now consider the possibilities for wholesale price discrimination. The manufacturer's profit function if she deviates in terms of w_H and w_L (to one low cost retailer) and retailers react to these deviations by setting p_H and p_L (to be determined later) is:

$$\pi(w_L, w_H) = \frac{1}{N}\left(1 + \frac{1}{(N-1)}G\left(\int_{p_L}^{p_H} D(p)dp\right) - G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)\right) w_L D(p_L(w_L))$$
$$+ \frac{N-2}{N}\left(1 + \frac{G\left(\int_{p_L^*}^{p_H} D(p)dp\right)}{(N-1)} + \frac{G\left(\int_{p_L^*}^{p_L} D(p)dp\right)}{(N-1)(N-2)} + \frac{G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)}{N-2}\right) w_L^* D(p_L^*(w_L^*))$$
$$+ \frac{1}{N}\left(1 - G\left(\int_{p_L^*}^{p_H} D(p)dp\right)\right) w_H D(p_H(w_H)).$$

This expression can be understood as follows. First, the term $\frac{1}{N}G\left(\int_{p_L^*}^{p_H} D(p)dp\right)$ in the last line is the share of consumers that first saw p_H and continue to search as they believe that all other firms choose p_L^*. The remaining of these consumers buy at the price p_H. Each of the other retailers gets $1/(N-1)$ of the consumers that continue to search. Retailers charging p_L^* will sell to these consumers, while a retailer that charges p_L will only get a fraction of these consumers, namely those with relatively higher search cost. Since they still believe that the other retailers charge p_L^*, all consumers with a search cost smaller than $G\left(\int_{p_L^*}^{p_L} D(p)dp\right)$ continue searching for the remaining retailers and buy there. Finally, there is a share of consumers that on their first search observes p_L and they continue to search if their search cost is smaller than $\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp$.

The first-order condition for the manufacturer with respect to w_H should be satisfied with equality. As in the previous section, the reason is that in an equilibrium with wholesale price discrimination, a fraction $G(\widehat{s})$ of consumers continues to search if observing p_H^* so that both upward and downward deviations in w_H (and subsequently in p_H) affect demand. At w_L^*, however, only upward deviations can be profitable: as consumers will only find out about the deviations once they have visited the retailer in question, downward deviations in retail price (and thus in wholesale prices) do not attract additional demand making such deviations always unprofitable.

In the proof of the Proposition below we show that the first-order conditions with respect to w_L and w_H evaluated at the equilibrium wholesale prices yield

$$w_L^* D'(p_L^*(w_L))\frac{\partial p_L}{\partial w_L} + D(p_L^*) \leq 0, \tag{10}$$

and

$$(1 - G(\widehat{s}))\left[w_H^* D'(p_H^*)\frac{\partial p_H}{\partial w_H} + D(p_H^*)\right] + g(0)D(p_H^*)\frac{\partial p_H}{\partial w_H}\left[w_L^* D(p_L^*) - w_H^* D(p_H^*)\right] = 0, \tag{11}$$

where the expressions for $\frac{\partial p_L}{\partial w_L}$ and $\frac{\partial p_H}{\partial w_H}$ are given in the appendix. Note that, as with uniform pricing, these expressions for the retail price reactions are different from the ones under commitment as consumers do not observe the deviation of the manufacturer and thus cannot react to it. Note also that (10) implies that the manufacturer does not have an incentive to deviate to multiple or even all low-cost retailers.

Apart from these first-order conditions, we also need to guarantee that the manufacturer does not have an incentive to give all retailers the same wholesale price, whether it is w_L^* or w_H^*. In principle, the manufacturer could set w_L^* or w_H^* to all retailers without any retailer noticing it at their price setting stage. To make such deviations unprofitable, we have to have that the manufacturer makes equal profits over the low and high cost retailers, thus we need:

$$w_H^* D(p_H^*) = w_L^* D(p_L^*) \tag{12}$$

in any equilibrium with wholesale price discrimination. Given (12) the first-order condition with respect to w_H can be simplified to

$$w_H^* D^{'}(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) = 0. \tag{13}$$

The next Proposition shows that without commitment there does not exist an equilibrium with wholesale price discrimination.

Proposition 7 *Without commitment, an equilibrium with wholesale price discrimination requires that the equations (4), (5), (12) and (13) and the inequality (10) are satisfied. If $\overline{s}$ is small enough, these requirements cannot be satisfied.*

The proof of the proposition basically shows that the only way to satisfy the equal profit condition (12) and not to have an incentive to set a different high wholesale price ((13) is satisfied) is for the manufacturer to set a low wholesale price w_L^* for which it has an incentive to deviate. Alternatively, the only way to guarantee that (10) is satisfied is if $w_H^* D(p_H^*) < w_L^* D(p_L^*)$. However, given that retailers do not observe the wholesale prices set to their competitors, the manufacturer would then be able to profitably and secretly deviate and set w_L^* to all retailers. Figure 7 shows that for linear demand if, together with (4), (5) and (13), (10) is satisfied with equality, then $w_H^* D(p_H^*) < w_L^* D(p_L^*)$ for any value of $\overline{s}$. Non-existence of an equilibrium with wholesale price discrimination is thus not only an issue for small enough values of $\overline{s}$.

Note that, unlike the argument on opportunism (see, e.g., McAfee and Schwartz [1994] and Rey and Vergé [2004]), the reason for the non-existence result in our context is *not* because we have assumed passive beliefs. An equilibrium with uniform pricing exists and the difference between unilateral and multilateral deviations that underlies the opportunism argument is not present here. The profitable deviation that is preventing equilibrium existence, not directly related to out-of-equilibrium beliefs at all, is the unilateral deviation where the manufacturer gives the retailer that is supposed to get a higher wholesale price the same (lower) wholesale equilibrium price as all the other retailers.

Numerical analysis, reported in the online Appendix, show that the conclusion of Proposition 7 also holds true when the search cost distribution is uniform and $\overline{s}$ is not

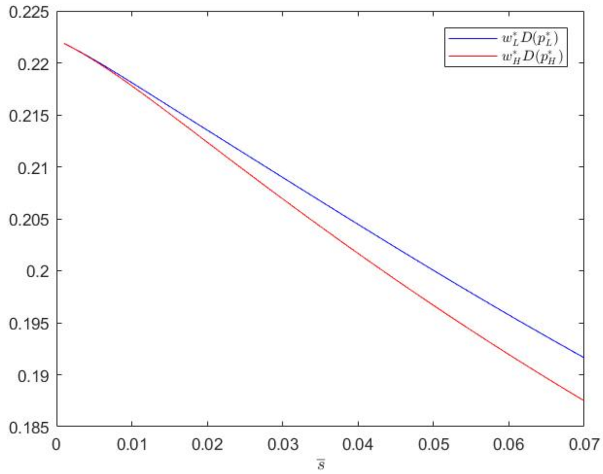


Figure 7: Manufacturer profit over the low and high cost retailers for different values of $\overline{s}$

small. Furthermore, there we also plot the necessary equilibrium condition (10) for the demand $D(p) = (1-p)^{\beta}$ and different values of β, given that the other equilibrium conditions are satisfied and show that the conclusion of the proposition continues to hold also then.

6 Requiring Sales at Recommended Price

RRPs are defined as non-binding suggestions made by manufacturers. In practice, it has been documented, however, that retailers often come up with false recommendations of this nature in order to influence the purchasing decision of consumers.[23] By seeing prices below the RRP, a consumer may be tempted to buy and not continue to search, enabling retailers to increase their margins. These practices are labelled as "fictitious pricing" by the Federal Trade Commission (FTC). In this section, we focus on the effect of the regulation imposed by the U.S. Code of Federal Regulations that requires that at least some sales have to take place at RRPs. This regulation acknowledges that many consumers believe that RRPs are prices at which products are generally sold.

In this paper, we stay closer to the definition of RRPs that are set by manufacturers. The regulation also addresses manufacturers' actions by claiming that in order for a manufacturer not to be chargeable with having participated in fictitious pricing it should suggest list prices by making an honest estimation of the actual retail price and make sure that at least some sales take place at the RRP. We show that by requiring that at least some sales take place at the RRP, the Code of Federal Regulations effectively resolves the issue of the non-existence of an equilibrium with wholesale price discrimination and allows the manufacturer to use RRPs to commit to wholesale price discrimination as follows.

[23]The New York Times reports that in many cases retailers simply come up with RRPs on their own and provides many examples (see, https://www.nytimes.com/2016/03/06/technology/its-discounted-but-is-it-a-deal-how-list-prices-lost-their-meaning.html)

The manufacturer announces the high retail price p_H^* as an RRP. She is then effectively committed to at least one retailer selling at this price and therefore has to choose w_H^* such that the retailer optimally reacts by setting p_H^*. Other retailers get a lower wholesale price w_L^* and sell at a price below the RRP. The deviation that destroyed the equilibrium with wholesale price discrimination (namely the manufacturer secretively setting the wholesale price w_L^* to all retailers) is penalized by the regulation and therefore not optimal any more. As the remaining equilibrium conditions (4), (5), (10) and (13) imply that $w_H^* D(p_H^*) < w_L^* D(p_L^*)$, the manufacturer is not tempted to set w_H^* to more than one retailer. Note that the observation that recommendations often do not bind in practice as most products sell at a price below the RRP naturally follows from our framework.

The next Proposition argues that the efficient equilibrium prices under wholesale price discrimination converge to the efficient equilibrium prices in the uniform pricing case if $\overline{s} \to 0$. Moreover, the comparative statics with respect to $\overline{s}$ is such that in a neighbourhood of $\overline{s} = 0$, the lowest wholesale and retail prices behave as in the uniform pricing equilibrium, whereas the highest wholesale and retail price charged are higher. Thus, consumers are worse off because of wholesale price discrimination. Furthermore, a fraction of consumers with low search costs has to search to find the low retail price p_L^*, while under uniform pricing consumers pay lower retail prices without further search.

Proposition 8 *If $\overline{s}$ is small enough and $\partial p^M(w^M)/\partial w < 1$, and regulation exists requiring sales at the recommend retail price, then an equilibrium with effective wholesale price discrimination exists where the manufacturer announces p_H^* as the RRP. The most efficient of these equilibria converges to $p_L^* = w_L^* = p_H^* = w_H^*$, where $w_L^* = w_H^* = w^*$ solves $\frac{1}{2} w^* D'(w^*) + D(w^*) = 0$. Moreover, in a neighbourhood of $\overline{s} = 0$ the comparative statics with respect to $\overline{s}$ is such that*

$$\begin{aligned} \frac{dp_L^*}{d\overline{s}} &= -\frac{x}{D(p_H^*)}, \frac{dp_H^*}{d\overline{s}} = -\frac{1}{D(p_H^*)} \frac{xN-1}{N}, \\ \frac{dw_L^*}{d\overline{s}} &= -\frac{1+x}{D(p_H^*)} \; \textit{and} \; \frac{dw_H^*}{d\overline{s}} = -\frac{1}{D(p_H^*)} \frac{(1+x)N-2}{N}. \end{aligned}$$

For larger values of $\overline{s}$ we numerically solve for linear demand and see how the equilibrium evolves under wholesale price discrimination. From the Proposition it follows that in a neighbourhood of $\overline{s} = 0$ and $N = 3$, $x = 2/3$, so that $\frac{dp_L^*}{d\overline{s}} \approx -2$, $\frac{dw_L^*}{d\overline{s}} \approx -5$, $\frac{dp_H^*}{d\overline{s}} \approx -1, \frac{dw_H^*}{d\overline{s}} \approx -3$. Figure 8(Left) shows how wholesale and retail prices change for different values of $\overline{s}$. It is clear that wholesale and retail prices are decreasing in $\overline{s}$.

Figure 8(Right) shows the difference in consumer surplus under wholesale price discrimination and uniform pricing. From the figure we can see that the impact of wholesale price discrimination on consumer surplus can be quite large. For instance, for an upper bound of the search cost distribution of 0.04, consumer surplus under wholesale price discrimination decreases by approximately 5%.

The comparison of retail prices under wholesale price discrimination and uniform pricing is depicted in Figure 9(Left) for general values of $\overline{s}$. It is clear that under wholesale

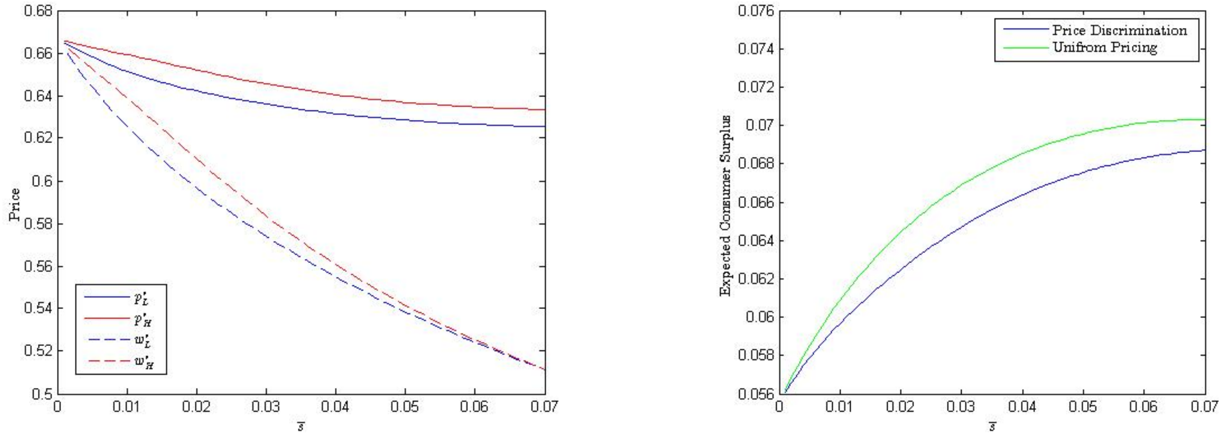


Figure 8: **Left:** Wholesale and Retail prices for different values of $\overline{s}$ and $N = 3$. **Right:** Expected Consumer Surplus for different values of $\overline{s}$ and $N = 3$.

price discrimination, both the low and the high retail prices are larger than the retail price under uniform pricing. The comparison between wholesale prices is depicted in Figure 9(Right) reinforcing Figure 9(Left) in that wholesale prices under wholesale price discrimination are larger than under uniform pricing.

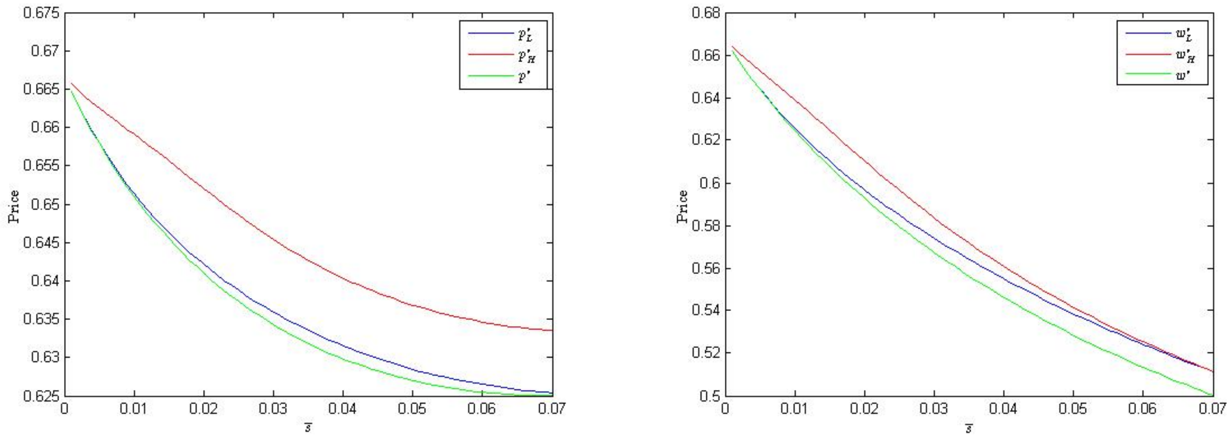


Figure 9: **Left:** Retail prices under uniform pricing and price discrimination and $N = 3$. **Right:** Wholesale prices under uniform pricing and price discrimination and $N = 3$.

Figure 10(Left) shows that both, low and high cost retailers, have lower margins under wholesale price discrimination. As argued before, wholesale price discrimination acts as a mechanism that indirectly screens searching consumers: consumers with different search costs react differently to retail prices inducing more competition between retailers. Figure 10(Right) shows the difference in retail profits between uniform pricing and wholesale price discrimination. Despite the lower margins, low cost retailers earn higher profits compared to a retailer under uniform pricing for smaller values of $\overline{s}$. The reason is that the difference in margins is small, while low cost retailers gain more sales due to low cost

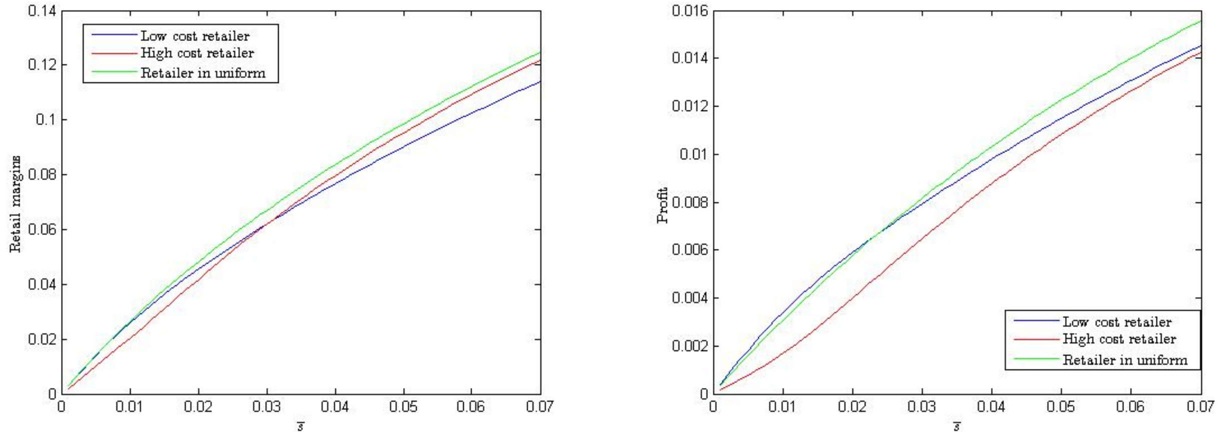


Figure 10: **Left:** Retail margins for different values of $\overline{s}$ and $N = 3$. **Right:** Retailers' Profit for different values of $\overline{s}$ and $N = 3$.

searchers that first visited the high cost retailer and then continued to search for the low cost retailers. From Propositions 6 and 8 it follows that this is actually a general result for small values of $\overline{s}$: the first-order approximation for retail margins of the low cost retailers under wholesale price discrimination are equal to the ones under uniform pricing, but under price discrimination each of these retailers gets a share of $\frac{1}{N}\left(1+\frac{1}{N(N-1)}\right)$ of the consumers, while under uniform pricing each retailer gets a share of $\frac{1}{N}$ of the consumers. For larger values of $\overline{s}$, the numerical analysis shows that it is the lower margins that dominate the impact on the low cost retailers' profits. The profit of retailers under uniform pricing are always higher than the profit the high cost retailer makes under wholesale price discrimination. Finally, we confirm in Figure 11 for $N = 3$ that the manufacturer earns higher profit under wholesale price discrimination and that the difference is increasing in $\overline{s}$. Similar figures obtained for larger values of N are provided in the online appendix.

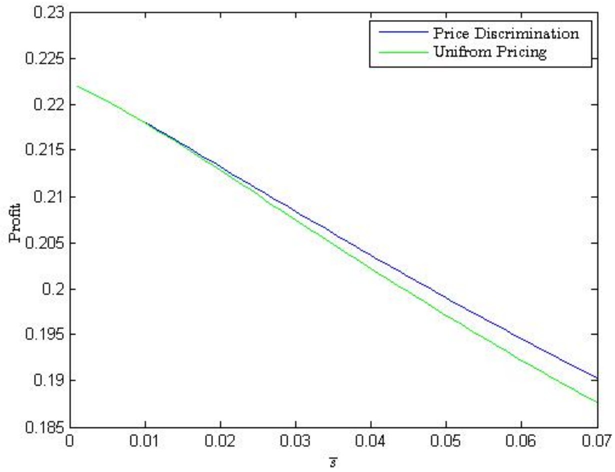


Figure 11: Manufacturer's Profit for different values of $\overline{s}$ and $N = 3$

7 Discussion and Conclusion

In this paper, we have focused on vertically related industries where consumers in the retail market have heterogeneous search cost. We have shown that the manufacturer has an incentive to set different wholesale prices to different retailers in order to stimulate consumers to search for lower prices, indirectly screen them according to their search costs, thus inducing more competition between retailers resulting in lower retail margins. If the manufacturer can commit to wholesale prices, we show that she will make more profit by discriminating between retailers, making both retailers and consumers worse off. Without commitment an equilibrium with wholesale price discrimination does not exist. In that case, legislation requiring that a substantial number of sales are made at RRPs gives manufacturers the possibility to partially commit allowing for an equilibrium with wholesale price discrimination accompanied by an announcement that the retail price of the high cost retailer(s) is the RRP. Despite the fact that competition authorities impose such restrictions with the aim of protecting consumers, we have shown that when RRPs are induced by manufacturers they actually may have the opposite effect. The fact that under wholesale price discrimination consumers are worse off despite it inducing more retail competition, sheds a new perspective on the economic rationale for the Robinson-Patman Act that only forbids price discrimination if it "lessens competition".

The price discrimination literature until now has firms differentiating between consumers with different valuations. In this paper, we have focussed on a very different function of price discrimination, namely to indirectly screen consumers with different search cost. In our story, it is essential that (i) consumers *believe* that some retailers have lower prices than others because they contract at a lower wholesale price, but do not know which retailer has which wholesale (or retail) price, and that (ii) consumers differ in their search cost. This is enough to induce more retail competition, lower retail margins and higher manufacturer profits. For (i) to be true, it must be that either retailers cannot effectively advertise their prices to consumers, or that a minimum advertised price (MAP) is in place forbidding retailers to advertise low retail prices (see Asker and Bar-Isaac [2016]).

We have focussed on a specific form of price discrimination where one retailer is charged a higher wholesale price and all the other retailers are charged a lower price. In an online Appendix, we show that qualitatively our main result continues to hold if the manufacturer extracts a part of the retail profits in terms of a fixed fee. Alternatively, a manufacturer may want to engage in other forms of price discrimination. For example, she could charge more than two different prices to retailers. What the optimal number of different wholesale prices is, is left for future research. In the online Appendix, we provide some thoughts on another issue, namely whether or not it is optimal to charge more than one retailer the highest wholesale price even if only two wholesale prices are charged.

The mechanism we unravelled may also affect other non-price aspects of the vertical relationship between manufacturers and retailers and we think that it is worthwhile in future research to see on which issues that are governed in contractual arrangements, manufacturers may induce asymmetries between retailers to induce more retail competition and when this may benefit or harm consumers.

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8 Appendix: Proofs

Proof of Proposition 2: To prove the existence and uniqueness of an equilibrium for $\overline{s}$ small enough, we prove that the retailers' and manufacturer's profit functions are concave. The second-order derivative of the retailers' profit function equals

$$-2g(0)D(p^*)D^{'}(p^*)(p^*-w)-g(0)D^2(p^*)+\Big(D^{''}(p^*)(p^*-w)+2D'(p^*)\Big).$$

It is clear that for $g(0)$ large enough and p^* being close to w^* this expression is negative. The second-order derivative of the manufacturer's profit function equals

$$2D'(p^*)\frac{\partial p^*}{\partial w}+wD''(p^*)\frac{\partial p^*}{\partial w}+wD'(p^*)\frac{\partial^2 p^*}{\partial w^2}$$

To show that this expression is also negative for $\overline{s}$ small enough, we first establish that in a neighbourhood of $\overline{s}=0$ $\frac{\partial^2 p^*}{\partial w^2}$ can be approximated as

$$\frac{2g^2(0)D^3(p^*)D'(p^*)\frac{\partial p^*}{\partial w}-2g^2(0)D^3(p^*)D'(p^*)\left(2\frac{\partial p^*}{\partial w}-1\right)}{g^2(0)D^4(p^*)}=-\frac{2D'(p^*)\left(\frac{\partial p^*}{\partial w}-1\right)}{D(p^*)},$$

which in a neighbourhood of $\overline{s}=0$ is approximately equal to 0. Thus, in a neighbourhood of $\overline{s}=0$ the second-order derivative of the manufacturer's profit function has the sign of $2D'(p^*)+p^*D''(p^*)$, which is negative given the conditions we imposed on the demand functions.

We now discuss the comparative static result. Given the expression for $\frac{\partial p^*}{\partial w}$ the manufacturer's first-order condition can be written as

$$0=wD'(p^*)\left(\frac{D'(p^*)}{g(0)}-D^2(p^*)\right)-2D^2(p^*)D^{'}(p^*)(p^*-w)-D^3(p^*)+\frac{D(p^*)D^{''}(p^*)(p^*-w)}{g(0)}+\frac{2D(p^*)D'(p^*)}{g(0)}.$$

Taking the total differential evaluated in a neighbourhood of $\overline{s}=0$ gives

$$0=\left(2D(p^*)D'(p^*)+wD'^2(p^*)\right)d\frac{1}{g(0)}+D'(p^*)D^2(p^*)dw,$$

which, using $D(p^*)+wD'(p^*)=0$, gives $dw=-\frac{1}{D(p^*)}d\frac{1}{g(0)}$.

Taking the total differential of the first-order condition (1) of the retailer and evaluating it in a neighbourhood of $\overline{s}=0$ where $g(0)\to\infty$ gives $d\frac{1}{g(0)}+D(p^*)dw^*-D(p^*)dp^*=0$. Substituting $dw=-\frac{1}{D(p^*)}d\frac{1}{g(0)}$ yields $dp^*=0$.

Proof of Proposition 3: We first establish that for small enough values of $\overline{s}$ (4) and (5) define the retail reactions for any w_L and w_H for the out-of-equilibrium beliefs specified in a neighbourhood of p_L^* and p_H^*. These reactions are indeed well-defined for small enough values of $\overline{s} = 0$ if the retail profit functions are concave as in that case the first-order conditions define global maxima. Taking, for instance, the second-order derivative of the profit function of the high cost retailer with respect to p_H yields

$$\begin{aligned} \frac{\partial^2 \pi_H}{\partial p_H^2} &= \frac{1-G(\widehat{s})}{N} \left[(p_H - w_H) D''(p_H) + 2D'(p_H) \right] - \frac{g(\widehat{s})}{N} D(p_H) \left[(p_H - w_H) D'(p_H) + D(p_H) \right] \\ & - \frac{g(\widehat{s})}{N} D(p_H) \left[2(p_H - w_H) D'(p_H) + D(p_H) \right] - \frac{g'(\widehat{s})}{N} (p_H - w_H) D^2(p_H). \end{aligned}$$

As in a neighbourhood of $\overline{s} = 0$ the relevant prices[24] are such that $(p_H - w_H) \approx 0$ this second-order derivative has the sign of

$$\frac{\partial^2 \pi_H}{\partial p_H^2} = 2 \frac{1-G(\widehat{s})}{N} D'(p_H) - 2 \frac{g(\widehat{s})}{N} D^2(p_H),$$

which is clearly negative. This shows that the high cost retailer does not want to deviate in a neighbourhood of p_H^* where we have fixed the out-of-equilibrium beliefs. Note, however, that for deviations outside the neighbourhood, the out-of-equilibrium beliefs are different. Below we will argue that we can specify continuous out-of-equilibrium beliefs such that the first-order condition for the high cost retailers indeed yields the global maximum.

For the low cost retailer's profit function, we can also show that the second-order derivative is negative given the out-of-equilibrium beliefs specified in the neighbourhood of p_L^*. For the low cost retailers this also implies that the first-order condition yields the global maximum for any other out-of-equilibrium belief specified outside the neighbourhood of p_L^*. The reason is that the belief that is specified in the neighbourhood of p_L^* yields the largest demand for a deviating retailer and any other belief yields lower expected deviation profits.

Next, we show how retail prices react to changes wholesale prices, i.e., we evaluate the respective different partial derivatives, in the neighbourhood of $\overline{s} = \frac{1}{g(0)} = 0$. Rewriting the retail first-order conditions (4) and (5) as

$$- D^2(p_H^*)(p_H^* - w_H) + \frac{(1 - G(\widehat{s}))}{g(\widehat{s})} \left[D'(p_H^*)(p_H^* - w_H) + D(p_H^*) \right] = 0, \tag{14}$$

and

$$- \left(\frac{(N-1)^2}{N} + 1 \right) D^2(p_L^*)(p_L^* - w_L) + \frac{((N-1) + G(\widehat{s}))}{g(\widehat{s})} \left[D'(p_L^*)(p_L^* - w_L) + D(p_L^*) \right] = 0, \tag{15}$$

[24] If p_H is so large that $\int_{p_L}^{p_H} D(p) dp > \overline{s}$ it is clear that the high cost retailer's profit equals 0 and these deviations are left out of consideration.

we the total differential of (14) yields $-D^2(p_H^*)\left(dp_H^*-dw_H\right)-D(p_H^*)\left(D(p_H^*)dp_H^*-D(p_L^*)dp_L^*\right)+D(p_H^*)d\frac{1}{g(\widehat{s})}=0$, or

$$-2dp_H^*+dw_H+dp_L^*+d\frac{1}{D(p_H^*)g(\widehat{s})}=0.$$

Similarly, taking the total differential of (15) and leaving out "irrelevant" terms we obtain $-\left(\frac{(N-1)^2}{N}+1\right)D^2(p_L^*)\left(dp_L^*-dw_L\right)+D(p_L^*)\left(D(p_H^*)dp_H^*-D(p_L^*)dp_L^*\right)+(N-1)D(p_L^*)d\frac{1}{g(\widehat{s})}=0$, or

$$-\frac{N^2+1}{N}dp_L^*+\frac{N^2-N+1}{N}dw_L+dp_H^*+d\frac{N-1}{D(p_L^*)g(\widehat{s})}=0.$$

Thus, the total effects of $w_L$and w_H on retail prices can be calculated by substituting these two equations into each other:

$$\frac{2N^2-N+2}{N}dp_L^*=2\frac{N^2-N+1}{N}dw_L+dw_H+(2N-1)\,d\frac{1}{D(p_L^*)g(\widehat{s})},$$

or

$$\left(2N^2-N+2\right)dp_L^*=2\left(N^2-N+1\right)dw_L+Ndw_H+d\frac{N\,(2N-1)}{D(p_L^*)g(\widehat{s})},\tag{16}$$

and

$$-2\frac{N^2+1}{N}dp_H^*+\frac{N^2+1}{N}dw_H+\frac{N^2+1}{N}d\frac{1}{D(p_H^*)g(\widehat{s})}+\frac{N^2-N+1}{N}dw_L+dp_H^*+d\frac{N-1}{D(p_L^*)g(\widehat{s})}=0$$

or

$$\left(2N^2-N+2\right)dp_H^*=\left(N^2+1\right)dw_H+\left(N^2-N+1\right)dw_L+d\frac{2N^2-N+1}{D(p_L^*)g(\widehat{s})}.\tag{17}$$

Thus, these equations give the unique equilibrium retail price reactions to $w_L$and w_H in a neighbourhood of $\overline{s}=0$. It follows that

$$\begin{aligned}\frac{\partial p_H^*}{\partial w_L^*}-\frac{\partial p_L^*}{\partial w_L^*}&=\frac{(N^2-N+1)-2\,(N^2-N+1)}{(2N^2-N+2)}=\frac{-N^2+N-1}{(2N^2-N+2)}<0\\ \frac{\partial p_H^*}{\partial w_H^*}-\frac{\partial p_L^*}{\partial w_H^*}&=\frac{(N^2-N+1)}{(2N^2-N+2)}>0.\end{aligned}$$

We now turn to the manufacturer's profit maximization problem. The first-order conditions with respect to w_L and w_H yield

$$\begin{aligned}0&=(w_LD(p_L^*)-w_HD(p_H^*))\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right)\\&\quad+\frac{[1-G(\widehat{s})]}{g(\widehat{s})}\left[D(p_H^*)+w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right]+\frac{N-1+G(\widehat{s})}{g(\widehat{s})}w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*},\end{aligned}$$

and

$$\begin{aligned}0&=(w_LD(p_L^*)-w_HD(p_H^*))\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}-D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right)\\&\quad+\frac{N-1+G(\widehat{s})}{g(\widehat{s})}\left[D(p_L^*)+w_LD'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right]+\frac{[1-G(\widehat{s})]}{g(\widehat{s})}w_HD'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}.\end{aligned}$$

In a neighbourhood of $\overline{s} = 0$ where $w_L D(p_L^*) = w_H D(p_H^*)$ and $g(\widehat{s})$ is large, the second-order conditions with respect to w_L and w_H have the sign of

$$D(p_H^*)\left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right)\left[-2D(p_H^*) - 2w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} + 2w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right],$$

and

$$D(p_L^*)\left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right)\left[2D(p_H^*) + 2w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} - 2w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right],$$

respectively. As $\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*} > 0$, while $\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*} < 0$, it is clear that both expressions are negative. Thus, the first-order conditions represent global maxima.

We now turn to the comparative statics results. The total differential of the first first-order condition in the neighbourhood of $\overline{s} = \frac{1}{g(0)} = 0$ where $wD'(p) \approx -D(p)$ and $D(p_L^*) \approx D(p_H^*)$ can be written as

$$\begin{aligned}
&\left(D(p_L^*) + w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} - w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} - D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right) dw_L - \\
&\left(D(p_H^*) - w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*} + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} - D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right) dw_H - \\
&\left(D(p_H^*) + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} - w_H D'(p_H^*)\frac{\partial p_L^*}{\partial w_H^*}\right) D(p_L^*)\left(\left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right) dw_H + \left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right) dw_L\right) \\
&+ \left(\left[D(p_H^*) + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right] + [N-1]\, w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*}\right) d\frac{1}{g(\widehat{s})} = 0,
\end{aligned}$$

or

$$\begin{aligned}
&\left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right)\left(\left(1 + \frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right) dw_L - \left(1 - \frac{\partial p_H^*}{\partial w_H^*} + \frac{\partial p_L^*}{\partial w_H^*}\right) dw_H\right) \\
&- \left(1 - \left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right)\right)\left(\left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right) dw_H + \left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right) dw_L\right) \\
&+ \left(\left[1 - \frac{\partial p_H^*}{\partial w_H^*}\right] - [N-1]\frac{\partial p_L^*}{\partial w_H^*}\right) d\frac{1}{D(p_H^*)g(\widehat{s})} = 0.
\end{aligned} \tag{18}$$

The total differential of the second first-order condition in the neighbourhood of $\overline{s} = \frac{1}{g(0)} = 0$ where $wD'(p) = -D(p)$ can be written as

$$\begin{aligned}
&\left(D(p_L^*) + w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} - w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*} - D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right) dw_L - \\
&\left(D(p_H^*) - w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*} + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*}\right)\left(D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*} - D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right) dw_H + \\
&\left(D(p_L^*) + w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} - w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right) D(p_L^*)\left(\left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right) dw_L + \left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right) dw_H\right) + \\
&\left((N-1)\left[D(p_L^*) + w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*}\right] + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*}\right) d\frac{1}{g(\widehat{s})} = 0,
\end{aligned}$$

or

$$\begin{aligned}
&\left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right)\left(\left(1 + \frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right) dw_L - \left(1 - \frac{\partial p_H^*}{\partial w_H^*} + \frac{\partial p_L^*}{\partial w_H^*}\right) dw_H\right) \\
&+ \left(1 - \left(\frac{\partial p_L^*}{\partial w_L^*} - \frac{\partial p_H^*}{\partial w_L^*}\right)\right)\left(\left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}\right) dw_L + \left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}\right) dw_H\right) \\
&+ \left(N\left[1 - \frac{\partial p_L^*}{\partial w_L^*}\right] - \left(1 - \left(\frac{\partial p_L^*}{\partial w_L^*} - \frac{\partial p_H^*}{\partial w_L^*}\right)\right)\right) d\frac{1}{D(p_L^*)g(\widehat{s})} = 0.
\end{aligned} \tag{19}$$

and therefore (18) can be simplified as

$$
\begin{aligned}
&\frac{N^2-N+1}{(2N^2-N+2)}\left(\left(1+\frac{-N^2+N-1}{(2N^2-N+2)}\right)dw_L-\left(1-\frac{(N^2-N+1)}{(2N^2-N+2)}\right)dw_H\right)\\
&-\left(1-\frac{(N^2-N+1)}{(2N^2-N+2)}\right)\left(\frac{(N^2-N+1)}{(2N^2-N+2)}dw_H+\frac{-N^2+N-1}{(2N^2-N+2)}dw_L\right)\\
&+\left(\left[1-\frac{N^2+1}{2N^2-N+2}\right]-[N-1]\frac{N}{2N^2-N+2}\right)d\frac{1}{D(p_H^*)g(\widehat{s})}=0,
\end{aligned}
$$

or

$$
2\frac{N^2-N+1}{2N^2-N+2}\left(N^2+1\right)(dw_L-dw_H)+d\frac{1}{D(p_H^*)g(\widehat{s})}=0,
$$

In addition, (19) can be simplified to the same expression. Substituting into (16) and (17) yields

$$
dp_L^*-dw_L^*=\left(\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)}+\frac{N\left(2N-1\right)}{(2N^2-N+2)}\right)d\frac{1}{D(p_L^*)g(\widehat{s})}, \tag{20}
$$

and

$$
dp_H^*-dw_H^*=\left(-\frac{1}{2\left(N^2+1\right)}+\frac{2N^2-N+1}{2N^2-N+2}\right)d\frac{1}{D(p_L^*)g(\widehat{s})} \tag{21}
$$

Also, we can approximate the fraction of consumers that continue to search after visiting the high cost retailer, $G(\widehat{s})=\int_{p_L^*}^{p_H^*}D(p)dp/\overline{s}$, by

$$
D(p_H^*)\frac{dp_H^*-dp_L^*}{d\overline{s}}=-\frac{N^2-N+1}{2N^2-N+2}D(p_H^*)\frac{dw_L^*-dw_H^*}{d\overline{s}}+d\frac{1}{g(\widehat{s})}\frac{1}{(2N^2-N+2)},
$$

which can be rewritten as

$$
D(p_H^*)\left(\frac{dp_H^*}{d\overline{s}}-\frac{dp_L^*}{d\overline{s}}\right)=\frac{4N^2-N+4}{2(N^2+1)(2N^2-N+2)}<\frac{1}{N}.
$$

We can now show that the high cost retailer does not have an incentive to deviate to prices outside the neighbourhood of p_H^*. If he would deviate and set p_L^* his profits will be equal to $(p_L^*-w_H^*)D(p_L^*)$ and we first show that in a neighbourhood of $\overline{s}=0$ this is strictly smaller than his equilibrium profits $(1-G(\widehat{s}))(p_H^*-w_H^*)D(p_H^*)$. This is the case if, and only if,

$$
\frac{dp_L^*-dp_H^*}{d\overline{s}}+\frac{dp_H^*-dw_H^*}{d\overline{s}}<\frac{dp_H^*-dw_H^*}{d\overline{s}}(1-G(\widehat{s})),
$$

or $G(\widehat{s})\frac{dp_H^*-dw_H^*}{d\overline{s}}<\frac{dp_H^*-dp_L^*}{d\overline{s}}$, or $-\frac{1}{2(N^2+1)}+\frac{2N^2-N+1}{2N^2-N+2}<1$. This is certainly the case. By the same token, a deviation to a price p in the neighbourhood of p_L^* or any price smaller than p_L^* is not optimal. For any $p\in(p_L^*+\varepsilon,p_H^*-\varepsilon)$ (that is outside the immediate neighbourhoods of the equilibrium prices) we can write $p=\alpha(p_L^*+\varepsilon)+(1-\alpha)(p_H^*-\varepsilon)$ for some $\alpha\in(0,1)$ and define the following consumer out-of-equilibrium belief Pr(low cost retailer has deviated to price p) $=\alpha$. Given that the profit function of the high cost retailer (assuming any deviation is attributed to a high cost retailer) is concave and that the high cost retailer does not have an incentive to deviate to prices in the neighbourhood of p_L^* it follows that given these beliefs, the high cost retailer does not want to deviate to prices $p \in (p_L^* + \varepsilon, p_H^* - \varepsilon)$. If consumers blame high cost retailers for deviations to prices $p > p_H^*$, it is clear that these retailers also do not want to deviate upwards.

Finally, we establish that in a neighbourhood of $\overline{s} = 0$ the manufacturer makes more profit under wholesale price discrimination than under uniform pricing. The change in the optimal manufacturer profits $w_L D(p_L^*) + \frac{1-G(\widehat{s})}{N} \left(w_H D(p_H^*) - w_L D(p_L^*) \right)$ in a neighbourhood of $\overline{s} = 0$ equals

$$
\begin{aligned}
&\frac{N-1+G(\widehat{s})}{N} \left(\frac{dw_L^*}{d\overline{s}} D(p_L^*) + w_L^* D'(p_L^*) \frac{dp_L^*}{d\overline{s}} \right) + \\
&\frac{1-G(\widehat{s})}{N} \left(\frac{dw_H^*}{d\overline{s}} D(p_H^*) + w_H^* D'(p_H^*) \frac{dp_H^*}{d\overline{s}} \right) - \frac{g(\widehat{s})}{N} D(p_H^*) \left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) \left(w_H D(p_H^*) - w_L D(p_L^*) \right) \\
=& D(p_L^*) \left(\frac{dw_L^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) + \frac{1-G(\widehat{s})}{N} D(p_L^*) \left(\frac{dp_L^*}{d\overline{s}} - \frac{dw_L^*}{d\overline{s}} + \frac{dw_H^*}{d\overline{s}} - \frac{dp_H^*}{d\overline{s}} \right) \\
&- \frac{D^2(p_H^*)}{N} \left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) \left(\frac{dw_H^*}{d\overline{s}} - \frac{dp_H^*}{d\overline{s}} - \frac{dw_L^*}{d\overline{s}} + \frac{dp_L^*}{d\overline{s}} \right) \\
=& D(p_L^*) \left(\frac{dw_L^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) + \frac{D(p_H^*)}{N} \left(1 - G(\widehat{s}) - D(p_H^*) \left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) \right) \left(\frac{dp_L^*}{d\overline{s}} - \frac{dw_L^*}{d\overline{s}} + \frac{dw_H^*}{d\overline{s}} - \frac{dp_H^*}{d\overline{s}} \right) \\
=& D(p_L^*) \left(\frac{dw_L^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) + \frac{1}{N} \left(1 - 2D(p_H^*) \left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) \right) \left(D(p_H^*) \left(\frac{dw_H^*}{d\overline{s}} - \frac{dw_L^*}{d\overline{s}} \right) - D(p_H^*) \left(\frac{dp_H^*}{d\overline{s}} - \frac{dp_L^*}{d\overline{s}} \right) \right) \\
=& - \left(\frac{N}{2\left(N^2+1\right)\left(N^2-N+1\right)} + \frac{N\left(2N-1\right)}{\left(2N^2-N+2\right)} \right) + \left[\frac{N^2}{N^2+1} - \frac{2}{2N^2-N+2} \right] \left[\frac{1}{2(N^2-N+1)(2N^2-N+2)} \right].
\end{aligned}
$$

This expression is larger than -1, the change in manufacturer profit under uniform pricing, if and only if,

$$
-\frac{N}{2(N^2+1)(N^2-N+1)} + \left[\frac{N^2}{N^2+1} - \frac{2}{2N^2-N+2} \right] \left[\frac{1}{2(N^2-N+1)(2N^2-N+2)} \right] > \frac{-2}{(2N^2-N+2)}
$$

or

$$
\frac{N}{2\left(N^2+1\right)} < 1 - \frac{1}{2(N^2-N+1)(2N^2-N+2)},
$$

which is true as the LHS is decreasing in N, while the RHS is increasing in N and the inequality certainly holds for N equals 3.

Proof of Proposition 5: Apart from the expression for $\frac{\partial \widetilde{p}(w^*)}{\partial w}$ all the equilibrium conditions are explained in the main text. From (8) it follows that:

$$
\begin{aligned}
&- g' \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D^3(\widetilde{p})(\widetilde{p} - w) \frac{d\widetilde{p}}{dw} - 2g \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D(\widetilde{p}) D'(\widetilde{p})(\widetilde{p} - w) \frac{d\widetilde{p}}{dw} \\
&- g \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D(\widetilde{p}) \left(D'(\widetilde{p})(\widetilde{p} - w) + D(\widetilde{p}) \right) \frac{d\widetilde{p}}{dw} - g \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D^2(\widetilde{p}) (\frac{d\widetilde{p}}{dw} - 1) \\
&+ \left(1 - G \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) \right) \left(\left(D''(\widetilde{p})(\widetilde{p} - w) + D'(\widetilde{p}) \right) \frac{d\widetilde{p}}{dw} + D'(\widetilde{p}) (\frac{d\widetilde{p}}{dw} - 1) \right) = 0,
\end{aligned}
$$

or,

$$
\begin{aligned}
&- g' \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D^3(\widetilde{p})(\widetilde{p} - w) \frac{d\widetilde{p}}{dw} - 3g \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D(\widetilde{p}) D'(\widetilde{p})(\widetilde{p} - w) \frac{\partial \widetilde{p}}{\partial w} \\
&- g \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) D^2(\widetilde{p}) (2\frac{\partial \widetilde{p}}{\partial w} - 1) + \left(1 - G \left(\int_{p^*(w^*)}^{\widetilde{p}} D(p)dp \right) \right) \left(D''(\widetilde{p})(\widetilde{p} - w) \frac{\partial \widetilde{p}}{\partial w} + D'(\widetilde{p}) (2\frac{\partial \widetilde{p}}{\partial w} - 1) \right) = 0.
\end{aligned}
$$

Using the fact that we want to evaluate $\frac{d\widetilde{p}}{dw}$ at $w = w^*$ we can use (1) to get

$$-g^{'}(0)\,D^3(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}-3g(0)D(\widetilde{p})D^{'}(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}-g(0)D^2(\widetilde{p})(2\frac{d\widetilde{p}}{dw}-1)$$
$$+D^{''}(\widetilde{p})(\widetilde{p}-w)\frac{d\widetilde{p}}{dw}+D'(\widetilde{p})(2\frac{d\widetilde{p}}{dw}-1)=0,$$

which gives the expression in (9).

Proof of Proposition 6: The first part of the Proposition easily follows as the expression for $\frac{\partial\widetilde{p}(w^*)}{\partial w}$ reduces to $\frac{1}{2}$ if $g(0)\to\infty$. To show existence we first show that the manufacturer does not want to *increase* her wholesale price. In particular, we show that

$$D(\widetilde{p})+wD'(\widetilde{p})\frac{\partial\widetilde{p}}{\partial w}\leq 0\quad\text{ for all } w>w^*.$$

First, note that if the manufacturer deviates and sets a w to one or multiple retailers such that all consumers who visit these retailers continue to search, she cannot make more profit than in equilibrium. In the best case, if the manufacturer sticks to the wholesale equilibrium price for one retailer, she will make the same profit as in equilibrium, while if she deviates to all retailers, she will make less profit as the retailers will react by setting $\widetilde{p}=w$ and $wD(w)$ is decreasing in w for all $w>w^*$ (because $2D'(w)+wD''(w)<0$ and the equilibrium wholesale price is such that $\frac{1}{2}w^*D^{'}(w^*)+D(w^*)\leq 0$ and thus larger than the optimal price of an integrated monopolist).

Thus, consider deviations such that some consumers still buy from the retailer where the manufacturer has deviated. In this case, the above inequality holds certainly true if the derivative of the LHS with respect to w

$$2D'(\widetilde{p})\frac{\partial\widetilde{p}}{\partial w}+wD''(\widetilde{p})\left(\frac{\partial\widetilde{p}}{\partial w}\right)^2+wD'(\widetilde{p})\frac{\partial^2\widetilde{p}}{\partial w^2}<0\quad\text{ for all } w>w^*. \tag{22}$$

From (9) it follows that in a neighbourhood of $\overline{s}=0$ where $g(s)\to\infty$ $\frac{\partial\widetilde{p}}{\partial w}$ can be approximated by

$$\frac{d\widetilde{p}}{dw}=\frac{1}{2}+\frac{3D^{'}(\tilde{p})(\widetilde{p}-w)}{-4D(\widetilde{p})}>\frac{1}{2}.$$

As $\lim_{\overline{s}\to\infty}\frac{\partial\widetilde{p}}{\partial w}=\frac{1}{2}$, it must be the case that $\frac{\partial^2\widetilde{p}}{\partial w^2}>0$ for small enough values of $\overline{s}$. Thus, (22) holds true if $(2D'(\widetilde{p})+wD''(\widetilde{p}))\frac{\partial\widetilde{p}}{\partial w}<0$. This is certainly the case as $2D'(\widetilde{p})+wD''(\widetilde{p})\approx 2D'(pw)+wD''(w)<0$ for small enough values of $\overline{s}$ and $\frac{\partial\widetilde{p}}{\partial w}>0$.

We next show that the manufacturer does not want to *decrease* her wholesale price either. The only candidate deviation is to deviate to w^M. So, we have to compare the equilibrium profit $w^*D(p^*)$ to $w^MD(p^M(w^M))$. If $w^*\leq p^M(w^M)$, deviating downwards to w^M cannot be profitable as retailers would not react to such a deviation. So, consider $w^*>p^M(w^M)$. In that case $\frac{1}{2}p^M(w^M)D^{'}(p^M(w^M))+D(p^M(w^M))\geq 0$.[25] Combining this inequality with the FOC of the retail monopoly price $D(p^M(w^M))+(p^M(w)-w^M)D'(p^M(w))=0$ it follows that $p^M(w)-w^M>\frac{1}{2}p^M(w^M)$ or $p^M(w^M)>2w^M$. But this

[25] As $pD''(p)+2D'(p)<0$ it follows that the derivative of $\frac{1}{2}pD^{'}(p)+D(p)<0$.

contradicts the manufacturer's optimality condition of the double marginalization price $D(p^M(w^M)) + w^M D'(p^M(w))\frac{\partial p^M(w^M)}{\partial w} = 0$ if $\frac{\partial p^M(w^M)}{\partial w} > 1$ as $w^* > 2w^M$.

To establish that an equilibrium exists for small enough values of $\overline{s}$, we also have to consider the retailer's decision problem. It is clear that downward deviations are not optimal for the retailer as they do not attract new customers by doing so. From the retailer's profit function, it follows that for all $\widetilde{p} \geq p^*$ the first-order derivative equals

$$-g\left(\int_{p^*}^{\widetilde{p}} D(p)dp\right) D^2(\widetilde{p})(\widetilde{p}-w) + D^{'}(\widetilde{p})(\widetilde{p}-w) + D(\widetilde{p}),$$

while the second-order derivative equals:

$$-g'\left(\textstyle\int_{p^*}^{\widetilde{p}} D(p)dp\right) D^3(\widetilde{p})(\widetilde{p}-w) - g\left(\textstyle\int_{p^*}^{\widetilde{p}} D(p)dp\right) D(\widetilde{p})\left(2D^{'}(\widetilde{p})(\widetilde{p}-w) + D(\widetilde{p})\right) + D^{''}(\widetilde{p})(\widetilde{p}-w) + 2D^{'}(\widetilde{p}).$$

As $(\widetilde{p}-w)$ is close to 0 if $\overline{s}$ is small and as $g'(s) > -M$ this expression is smaller than 0 if $\overline{s}$ is small. Thus, for small enough values of $\overline{s}$ the profit function is concave and the retailers' FOC yields the global maximum.

To prove the comparative statics results, we first rewrite the equilibrium condition for the manufacturer in a neighbourhood of $\overline{s} = 0$ as

$$\begin{aligned} 0 &= wD'(p^*)\left(\frac{D'(p^*)}{g(0)} - D^2(p^*)\right) - 3D^2(p^*)D^{'}(p^*)(p^*-w) - 2D^3(p^*) \\ &+ \frac{2D'(p^*)D(p^*) + 2D^{''}(p^*)D(p^*)(p^*-w) - g^{'}(0)\, D^4(p^*)(p^*-w)}{g(0)}. \end{aligned}$$

Taking the total differential and taking into account that in a neighbourhood of $\overline{s} = 0, g(0) \to \infty$ this approximately yields

$$\begin{aligned} 0 &\approx D'(p^*)\left(w^*D'(p^*) + 2D(p^*)\right) d\frac{1}{g(0)} + 2D'(p^*)D^2(p^*)dw \\ &+ \left(-w^*D''(p^*)D^2(p^*) - 2w^*D'^2(p^*)D(p^*) - 9D^2(p^*)D^{'}(p^*)\right) dp^*. \end{aligned}$$

As $\frac{1}{2}w^*D'(p^*) + D(p^*) = 0$ the first term is approximately equal to 0 so that we have

$$dw^* = \frac{w^*D''(p^*) + 5D^{'}(p^*)}{2D'(p^*)} dp^*.$$

From the proof of Proposition 2, we know that the total differential of the first-order condition (1) of the retailer evaluated in a neighbourhood of $\overline{s} = 0$ is

$$d\frac{1}{g(0)} + D(p^*)dw^* - D(p^*)dp^* = 0.$$

Combining these two equations gives

$$\frac{dw^*}{d\frac{1}{g(0)}} = -\frac{w^*D''(p^*) + 5D^{'}(p^*)}{D(p^*)\left(w^*D''(p^*) + 3D^{'}(p^*)\right)}.$$

As the demand function satisfies $w^* D''(p^*) + 2D'(p^*) < 0$ it follows that both $\frac{dw^*}{d\frac{1}{g(0)}}$ and $\frac{dp^*}{d\frac{1}{g(0)}}$tare negative.

Proof of Proposition 7. In an equilibrium the FOCs for profit maximization for both retailers should be satisfied. For the high-cost retailer the FOC can be written as

$$-g\left(\widehat{s}+\int_{p_H^*}^{p_H} D(p)dp\right) D^2(p_H)(p_H - w_H) + \left(1 - G\left(\widehat{s}+\int_{p_H^*}^{p_H} D(p)dp\right)\right)\left[D'(p_H)(p_H - w_H) + D(p_H)\right]. \tag{23}$$

Taking the total differential gives

$$\begin{aligned}
&-3g\left(\widehat{s}+\int_{p_H^*}^{p_H} D(p)dp\right) D(p_H)D'(p_H)(p_H - w_H)\frac{dp_H}{dw_H} \\
&-g\left(\widehat{s}+\int_{p_H^*}^{p_H} D(p)dp\right) D^2(p_H)(2\frac{dp_H}{dw_H}-1) - g'\left(\widehat{s}+\int_{p_H^*}^{p_H} D(p)dp\right) D^3(p_H)(p_H - w_H)\frac{dp_H}{dw_H}+ \\
&\left(1 - G\left(\widehat{s}+\int_{p_H^*}^{p_H} D(p)dp\right)\right)\left[D''(p_H)(p_H - w_H)\frac{dp_H}{dw_H} + D'(p_H)(2\frac{dp_H}{dw_H}-1)\right] = 0,
\end{aligned}$$

which evaluated at the equilibrium values yields

$$\begin{aligned}
&-g'(\widehat{s})\, D^3(p_H^*)(p_H^* - w_H^*)\frac{dp_H}{dw_H} - 3g(\widehat{s})\, D(p_H^*)D'(p_H^*)(p_H^* - w_H^*)\frac{dp_H}{dw_H} - g(\widehat{s})\, D^2(p_H^*)(2\frac{dp_H}{dw_H}-1) \\
&+(1-G(\widehat{s}))\left[D''(p_H^*)(p_H^* - w_H^*)\frac{dp_H}{dw_H} + D'(p_H^*)(2\frac{dp_H}{dw_H}-1)\right] = 0.
\end{aligned}$$

Thus,

$$\frac{dp_H}{dw_H} = \frac{(1-G(\widehat{s}))D'(p_H^*) - g(\widehat{s})D^2(p_H^*)}{-g'(\widehat{s})D^3(p_H^*)(p_H^*-w_H^*) - 3g(\widehat{s})D(p_H^*)D'(p_H^*)(p_H^*-w_H^*) + (1-G(\widehat{s}))\left[D''(p_H^*)(p_H^*-w_H^*) + 2D'(p_H^*)\right] - 2g(\widehat{s})D^2(p_H^*)}.$$

Using the first-order condition (4), we can rewrite

$$\frac{dp_H}{dw_H} = \frac{-\frac{D(p_H^*)}{(p_H^*-w_H^*)}}{-\left(3D'(p_H^*) + \frac{g'(\widehat{s})}{g(\widehat{s})}\right)\left[\frac{D'(p_H^*)(p_H^*-w_H^*)}{D(p_H^*)}+1\right] + D''(p_H^*)(p_H^* - w_H^*) - \frac{2D(p_H^*)}{(p_H^*-w_H^*)}}. \tag{24}$$

For the low-cost retailer we can perform a similar analysis to evaluate $\frac{\partial p_L}{\partial w_L}$. Taking the first-order condition of (3) with respect to p_L yields

$$\begin{aligned}
0 \quad = \quad &\left[1 - G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{G\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{(N-1)}\right]\left[D'(p_L)(p_L - w_L) + D(p_L)\right] \\
&-\left(\frac{N-1}{N}g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right) + \frac{g\left(\int_{p_L}^{p_H^*} D(p)dp\right)}{N-1}\right) D^2(p_L)(p_L - w_L).
\end{aligned}$$

Taking the total differential and inserting equilibrium values gives

$$\begin{aligned}
0 \quad = \quad &-\left[\frac{N-1}{N}g(0) + \frac{g(\widehat{s})}{N-1}\right] D(p_L)\left[D'(p_L)(p_L - w_L) + D(p_L)\right]\frac{dp_L}{dw_L} + \\
&\left[1 + \frac{G(\widehat{s})}{(N-1)}\right]\left[D''(p_L)(p_L - w_L)\frac{dp_L}{dw_L} + D'(p_L)(2\frac{dp_L}{dw_L}-1)\right] \\
&-\left(\left(\frac{N-1}{N}\right)^2 g'(0) - \frac{g'(\widehat{s})}{N-1}\right) D^3(p_L)(p_L - w_L)\frac{dp_L}{dw_L} \\
&-\left(\frac{N-1}{N}g(0) + \frac{g(\widehat{s})}{N-1}\right) D(p_L)\left(2D'(p_L)(p_L - w_L)\frac{dp_L}{dw_L} + D(p_L)(\frac{dp_L}{dw_L}-1)\right),
\end{aligned}$$

which can be rewritten as

$$
\begin{aligned}
0 \;=\;& -3\left[\frac{N-1}{N}g\left(0\right)+\frac{g\left(\widehat{s}\right)}{N-1}\right]D(p_L)D'(p_L)(p_L-w_L)\frac{dp_L}{dw_L}+\\
&\left[1+\frac{G\left(\widehat{s}\right)}{(N-1)}\right]\left[D''(p_L)(p_L-w_L)\frac{dp_L}{dw_L}+D'(p_L)(2\frac{dp_L}{dw_L}-1)\right]\\
&-\left(\left(\frac{N-1}{N}\right)^2 g'\left(0\right)-\frac{g'\left(\widehat{s}\right)}{N-1}\right)D^3(p_L)(p_L-w_L)\frac{dp_L}{dw_L}\\
&-\left(\frac{N-1}{N}g\left(0\right)+\frac{g\left(\widehat{s}\right)}{N-1}\right)D^2(p_L)(2\frac{dp_L}{dw_L}-1),
\end{aligned}
$$

or

$$
\frac{dp_L}{dw_L}=\frac{\left[1+\frac{G(\widehat{s})}{(N-1)}\right]D'(p_L^*)-\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)D^2(p_L^*)}{-\left(\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}\right)D^3(p_L^*)(p_L^*-w_L^*)-\left[\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right]\left(3D(p_L^*)D'(p_L^*)(p_L^*-w_L^*)+2D^2(p_L^*)\right)+\left[1+\frac{G(\widehat{s})}{(N-1)}\right]\left[D''(p_L^*)(p_L^*-w_L^*)+2D'(p_L^*)\right]} \tag{25}
$$

Using the first-order condition (5) evaluated at equilibrium values, we can rewrite

$$
\frac{dp_L}{dw_L}=\frac{-\frac{D(p_L^*)}{(p_L^*-w_L^*)}}{-\left[\frac{D'(p_L^*)}{D(p_L^*)}(p_L^*-w_L^*)+1\right]\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)+D''(p_L^*)(p_L^*-w_L^*)-\frac{2D(p_L^*)}{p_L^*-w_L^*}}. \tag{26}
$$

From the expressions for $\frac{dp_H}{dw_H}$ and $\frac{dp_L}{dw_L}$ it follows that in a neighbourhood of $\overline{s}=0$ where $p_i^*\approx w_i^*, i=L,H$

$$
\frac{dp_L}{dw_L}-\frac{dp_H}{dw_H}=-\frac{\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)(p_H^*-w_H^*)}{4D(p_L^*)}+\frac{\left(3D'(p_H^*)+\frac{g'(\widehat{s})}{g(\widehat{s})}\right)(p_L^*-w_L^*)}{4D(p_H^*)}.
$$

We now prove that in a neighbourhood of $\overline{s}=0$ we have that if (13), then: $w_L^*D'(p_L^*)\frac{\partial p_L}{\partial w_L}+D(p_L^*)$

$$
\approx\quad w_L^*D'(p_L^*)\left(\frac{\partial p_H}{\partial w_H}-\frac{\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)(p_H^*-w_H^*)}{4D(p_L^*)}+\frac{\left(3D'(p_H^*)+\frac{g'(\widehat{s})}{g(\widehat{s})}\right)(p_L^*-w_L^*)}{4D(p_H^*)}\right)+D(p_L^*)>0.
$$

Our claim is true if

$$
\begin{aligned}
0 \;>\;& (w_H^*D'(p_H^*)-w_L^*D'(p_L^*))\frac{\partial p_H}{\partial w_H}+D(p_H^*)-D(p_L^*)+\\
& w_L^*D'(p_L^*)\left(\frac{\left(3D'(p_L^*)+\frac{\left(\frac{N-1}{N}\right)^2 g'(0)-\frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N}g(0)+\frac{g(\widehat{s})}{N-1}\right)}\right)(p_H^*-w_H^*)}{4D(p_L^*)}-\frac{\left(3D'(p_H^*)+\frac{g'(\widehat{s})}{g(\widehat{s})}\right)(p_L^*-w_L^*)}{4D(p_H^*)}\right).
\end{aligned}
$$

In a neighborhood of $\overline{s}=0$ we can write $w_i^*=w^*+dw_i, D(p_i^*)=D(p^*)+D'(p_i^*)dp_i^*$ and $D'(p_i^*)=D'(p^*)+D''(p_i^*)dp_i^*$, $i=L,H$. Thus, the first-order approximation of the right-hand side is

$$
\begin{aligned}
0 > {} & (D'(p^*)(dw_H - dw_L) + w^* D''(p^*)(dp_H - dp_L)) \frac{\partial p_H}{\partial w_H} + D'(p^*)(dp_H - dp_L) \\
& - w^* \frac{D'(p^*)}{4D(p^*)} \left(3D'(p^*) \left(dw_H - dw_L - (dp_H - dp_L) \right) - \frac{\left(\frac{N-1}{N}\right)^2 g'(0) - \frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N} g(0) + \frac{g(\widehat{s})}{N-1}\right)} (dp_H - dw_H) + \frac{g'(\widehat{s})}{g(\widehat{s})} (dp_L - dw_L) \right).
\end{aligned} \tag{27}
$$

From the equal profit condition $w_L^* D(p_L^*) = w_H^* D(p_H^*)$ it follows that $D(p^*)dw_L + w^* D'(p^*)dp_L = D(p^*)dw_H + w^* D'(p^*)dp_H$ so that using $\frac{1}{2} w^* D'(p^*) + D(p^*) = 0$ we have $dw_H - dw_L \approx 2(dp_H - dp_L)$. As $g(0) \to \infty$ when $\overline{s} \to 0$ and as $g'(s)$ is bounded so that $\frac{g'(\widehat{s})}{g(\widehat{s})}$ also approaches 0 if $\overline{s} \to 0$, we can rewrite (27) as

$$
\left(w^* D''(p^*) \frac{\partial p_H}{\partial w_H} + 2D'(p^*) - \frac{3}{4} \frac{w^* D'^2(p^*)}{D(p^*)} \right) (dp_H - dp_L) < 0.
$$

This clearly needs to be the case as in an equilibrium with wholesale price discrimination $dp_H - dp_L > 0$, whereas $w^* D''(p^*) \frac{\partial p_H}{\partial w_H} + 2D'(p^*) < 0$ because of the second-order condition for profit maximization.

Proof of Proposition 8. We first show that if an equilibrium exists, it must be that $\frac{1}{2} w^* D'(w^*) + D(w^*) = 0$ in the limit where $\overline{s} \to 0$. From (4) it is clear that in any equilibrium with wholesale price discrimination $p_H^* \to w_H^*$. As $0 < \widehat{s} < \overline{s}$, where $\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$, it must be the case that $p_H^* \to p_L^*$ if $\overline{s} \to 0$. Next, consider (5) if $\overline{s} \to 0$. Since also $\hat{s} \to 0$, and $D'(p_L^*) < 0$ while $D(p_L^*) > 0$ it must be that in any equilibrium with wholesale price discrimination $p_L^* \to w_L^*$. Thus, if $\overline{s} \to 0$ it follows that $p_H^* \approx p_L^* \approx w_H^* \approx w_L^*$. It remains to be seen to which values the wholesale and retail prices converge. Consider (10) and that (25) implies that $\frac{\partial p_L}{\partial w_L} \approx \frac{1}{2}$ in a neighbourhood of $\overline{s} = 0$ where $p_L^* - w_L^* \approx 0$ the first-order condition determining w_L^* can be simplified to $\frac{1}{2} w_L^* D'(w_L^*) + D(w_L^*) \approx 0$.

We now prove the comparative statics results assuming an equilibrium exists and come back to the existence issue at the end of the proof. Substituting (26), (10) can be written as

$$
\begin{aligned}
0 &= -w_L^* D'(p_L^*) D(p_L^*) + D''(p_L^*) 2D(p_L^*)(p_L^* - w_L^*)^2 - 2D^2(p_L^*) \\
& \quad - \left[D'(p_L^*)(p_L^* - w_L^*)^2 + D(p_L^*)(p_L^* - w_L^*) \right] \left(3D'(p_L^*) + \frac{\left(\frac{N-1}{N}\right)^2 g'(0) - \frac{g'(\widehat{s})}{N-1}}{\left(\frac{N-1}{N} g(0) + \frac{g(\widehat{s})}{N-1}\right)} \right).
\end{aligned}
$$

Taking the total differential in a neighbourhood of $\overline{s} = 0$, where $p_L^* \approx w_L^*$ and $g(0)$ and $g(\widehat{s})$ are large, gives

$$
-D(p_L^*) D'(p_L^*) dw_L^* - w_L^* \left(D(p_L^*) D''(p_L^*) + D'^2(p_L^*) \right) dp_L^* - 4D(p_L^*) D'(p_L^*) dp_L^* - 3D'(p_L^*) D(p_L^*) (dp_L^* - dw_L^*) \approx 0,
$$

which can be rewritten as

$$
2D'(p_L^*) dw_L^* - \left(w_L^* D''(p_L^*) + w_L^* \frac{D'^2(p_L^*)}{D(p_L^*)} + 7D'(p_L^*) \right) dp_L^* \approx 0.
$$

Thus, we have

$$dw_L^* \approx \left(\frac{w_L^* D''(p_L^*) + 5D'(p_L^*)}{2D'(p_L^*)} \right) dp_L^*. \tag{28}$$

As $g'(s)$ is bounded we can approximate $G(\widehat{s})$ in a neighbourhood of $\overline{s} = 0$ by $g(0) \int_{p_L^*}^{p_H^*} D(p)dp$ and approximate the first-order condition of the low-cost retailer as

$$0 \approx -\left(\frac{(N-1)^2}{N} + 1 \right) D^2(p_L^*)(p_L^* - w_L^*) + \int_{p_L^*}^{p_H^*} D(p)dp \left[D'(p_L^*)(p_L^* - w_L^*) + D(p_L^*) \right] + \frac{(N-1)\left[D'(p_L^*)(p_L^* - w_L^*) + D(p_L^*) \right]}{g(0)}.$$

Taking the total differential in a neighbourhood of $\overline{s} = 0$ gives

$$0 \approx -\left(\frac{(N-1)^2}{N} + 1 \right) D(p_L^*)(dp_L^* - dw_L^*) + (N-1)d\frac{1}{g(0)} + D(p_L^*)dp_H^* - D(p_L^*)dp_L^*. \tag{29}$$

Similarly, we can rewrite the first-order condition of the high-cost retailer as

$$-D^2(p_H^*)(p_H^* - w_H) - \left[D'(p_H^*)(p_H^* - w_H) + D(p_H^*) \right] \int_{p_L^*}^{p_H^*} D(p)dp + \frac{\left[D'(p_H^*)(p_H^* - w_H) + D(p_H^*) \right]}{g(0)} \approx 0.$$

Taking the total differential in a neighbourhood of $\overline{s} = 0$ gives

$$-D^2(p_H^*)(dp_H^* - dw_H) + D(p_H^*)d\frac{1}{g(0)} - D^2(p_H^*)dp_H^* + D(p_H^*)D(p_L^*)dp_L^* \approx 0,$$

or

$$-D(p_H^*)(2dp_H^* - dw_H) + d\frac{1}{g(0)} + D(p_H^*)dp_L^* \approx 0, \tag{30}$$

Finally, we consider the first-order condition of the manufacturer for the high-cost wholesale price

$$(1 - G(\widehat{s})) \left[w_H^* D'(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) \right] + g(0)D(p_H^*) \frac{\partial p_H}{\partial w_H} \left[w_L^* D(p_L^*) - w_H^* D(p_H^*) \right] = 0.$$

This can be approximated as

$$\left(\frac{1}{g(0)} - \int_{p_L^*}^{p_H^*} D(p)dp \right) \left[w_H^* D'(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) \right] + D(p_H^*) \frac{\partial p_H}{\partial w_H} \left[w_L^* D(p_L^*) - w_H^* D(p_H^*) \right] \approx 0,$$

so that the total differential in a neighbourhood of $\overline{s} = 0$ yields

$$w_L^* D'(p_L^*)dp_L^* + D(p_L^*)dw_L^* \approx w_H^* D'(p_H^*)dp_H^* + D(p_H^*)dw_H^*,$$

or, using $w_L^* D'(p_L^*)\frac{1}{2} + D(p_L^*) = 0$,

$$-2dp_L^* + dw_L^* \approx -2dp_H^* + dw_H^*, \tag{31}$$

Thus, we should solve the four equations (28), (29), (30) and (31) to solve for the respective derivatives. Combining (30) and (31) gives

$$D(p_H^*)(dp_L^* - dw_L^*) \approx d\frac{1}{g(0)}. \tag{32}$$

Combined with (28) gives

$$dp_L^* \approx -\frac{1}{D(p_H^*)}\frac{2D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)} d\frac{1}{g(0)},$$

and

$$dw_L^* \approx -\frac{1}{D(p_H^*)}\frac{w_L^* D''(p_L^*) + 5D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)} d\frac{1}{g(0)}$$

Substitute (32) into (29) gives

$$-\frac{1}{N}(dp_L^* - dw_L^*) + dp_H^* - dp_L^* \approx 0.$$

Combined with the expressions for dp_L^* and dw_L^* gives

$$dp_H^* \approx -\frac{1}{D(p_H^*)}\left(-\frac{1}{N} + \frac{2D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)}\right) d\frac{1}{g(0)}.$$

Substituting all expressions into (31) yields

$$\begin{aligned} dw_H^* &\approx 2\left(dp_H^* - dp_L^*\right) + dw_L^* \approx \frac{2}{N}\left(dp_L^* - dw_L^*\right) + dw_L^* \\ &\approx -\frac{1}{D(p_H^*)}\left(-\frac{2}{N} + \frac{w_L^* D''(p_L^*) + 5D'(p_L^*)}{w_L^* D''(p_L^*) + 3D'(p_L^*)}\right) d\frac{1}{g(0)}. \end{aligned}$$

This proves the comparative statics results.

Finally, we prove an equilibrium with wholesale price discrimination exists if $\overline{s}$ is small enough and $\frac{\partial p^M(w^M)}{\partial w} < 1$. The first part to notice is that the comparative statics results indeed show that $p_H^* > p_L^*$ and $w_H^* > w_L^*$ in a neighbourhood of $\overline{s} = 0$. Next, we will follow similar steps as in the proof of Proposition 2 but for both w_H^* and w_L^* separately to show that the manufacturer does not want to increase these respective wholesale prices beyond their equilibrium values. The part of the proof showing that the manufacturer does not want to decrease her wholesale prices are similar to the proof of Proposition 6, while the part showing that the retail profit functions are well-behaved are similar to the proof of Proposition 3 and will not be repeated here.

Like in the proof of Proposition 3 it is clear that the manufacturer does not want to increase its prices such that all consumers visiting that retailer will continue to search. In addition, in the range of prices where some consumers continue to buy from a retailer it suffices that the second-order derivative of the manufacturer's profit function with respect to $w_i, i = L, H$, is negative

$$2D'(\widetilde{p}_i)\frac{\partial \widetilde{p}_i}{\partial w_i} + w_i D''(\widetilde{p}_i)\left(\frac{\partial \widetilde{p}_i}{\partial w_i}\right)^2 + w_i D'(\widetilde{p}_i)\frac{\partial^2 \widetilde{p}_i}{\partial w_i^2} < 0 \quad \text{for } i = L, H \text{ and all } w > w^*.$$

From (24) it follows that in a neighbourhood of $\overline{s} = 0$ where $g(s) \to \infty$ $\frac{\partial \widetilde{p}_H}{\partial w_H}$ can be approximated by

$$\frac{\partial \widetilde{p}_H}{\partial w_H} \approx \frac{1}{2} + \frac{3D'(p_H^*)(p_H - w_H)}{-4D(p_H)} > \frac{1}{2}.$$

Similarly, in a neighbourhood of $\overline{s} = 0$ (26) can be approximated by

$$\frac{\partial \widetilde{p}_L}{\partial w_L} \approx \frac{1}{2} + \frac{3D'(p_L^*)(p_L - w_L)}{-2D(p_L)} > \frac{1}{2}.$$

Thus, we can argue that $\frac{\partial^2 \widetilde{p}_i}{\partial w_i^2} > 0, i = L, H$ in a neighbourhood of $\overline{s} = 0$. Therefore, the second-order condition is satisfied and the manufacturer does not want to increase her wholesale prices beyond their equilibrium values.

Like in the proof of Proposition 3 it is clear that the manufacturer does not want to increase its prices such that all consumers visiting that retailer will continue to search. In addition, in the range of prices where some consumers continue to buy from a retailer we need to show that the manufacturer does not want to deviate with one or multiple wholesale prices. First, we show that the second-order derivatives of the manufacturer's profit function with respect to $w_i, i = L, H$, are negative

$$2D'(\widetilde{p}_i)\frac{\partial \widetilde{p}_i}{\partial w_i} + w_i D''(\widetilde{p}_i)\left(\frac{\partial \widetilde{p}_i}{\partial w_i}\right)^2 + w_i D'(\widetilde{p}_i)\frac{\partial^2 \widetilde{p}_i}{\partial w_i^2} < 0 \quad \text{for } i = L, H \text{ and all } w > w^*.$$

From (24) it follows that in a neighbourhood of $\overline{s} = 0$ where $g(s) \to \infty$, $\frac{\partial \widetilde{p}_H}{\partial w_H}$ can be approximated by

$$\frac{\partial \widetilde{p}_H}{\partial w_H} \approx \frac{1}{2} + \frac{3D'(p_H^*)(p_H - w_H)}{-4D(p_H)} > \frac{1}{2}.$$

Similarly, in a neighbourhood of $\overline{s} = 0$ (26) can be approximated by

$$\frac{\partial \widetilde{p}_L}{\partial w_L} \approx \frac{1}{2} + \frac{3D'(p_L^*)(p_L - w_L)}{-2D(p_L)} > \frac{1}{2}.$$

Thus, we can argue that $\frac{\partial^2 \widetilde{p}_i}{\partial w_i^2} > 0, i = L, H$ in a neighbourhood of $\overline{s} = 0$ and that therefore, the second-order conditions are satisfied.

To show that the manufacturer does not want to deviate with multiple wholesale prices, we proceed in a few steps. First, it is clear that the manufacturer does not want to decrease w_L, as these retailers will not follow suit in lowering retail prices in response. Second, consider an increase in w_H and an increase in one or more w_L's. From the above it is clear that, keeping all w_L's at their equilibrium values, an increase in w_H cannot increase profits, despite the fact that $w_H^* D(p_H^*) < w_L^* D(p_L^*)$. As it follows from (10) that

$w_L D(p_L(w_L))$ is decreasing in w_L it cannot be the case that increasing w_H and one or more w_L's is profitable. Finally, and similar to the second step, one can argue that a decrease in w_H combined with an increase in one or more w_L's is not profitable.

Finally, we need to show that in the equilibrium $w_L^* D(p_L^*) > w_H^* D(p_H^*)$ so that the manufacturer does not want to set w_H^* to more firms (while the regulation requiring some sales to occur at p_H^* after announcing p_H^* as the RRP prevents the manufacturer to charge all firms w_L^*. This part of the proof relies heavily on the proof of the Proposition 7. First, from that proof we know that $w_L^* D(p_L^*)$ cannot be equal to $w_H^* D(p_H^*)$. Suppose then that $w_L^* D(p_L^*) < w_H^* D(p_H^*)$. From (11) it then follows that $w_H^* D^{'}(p_H^*) \frac{\partial p_H}{\partial w_H} + D(p_H^*) > 0$. We need to show that this implies that $w_L^* D^{'}(p_L^*) \frac{\partial p_L}{\partial w_L} + D(p_L^*) > 0$. We can follow the same steps as in the second part of the proof of Proposition 7. In particular, we can use (27) and use that from the hypothesis that $w_L^* D(p_L^*) < w_H^* D(p_H^*)$ in a neighbourhood of $\overline{s} = 0$ (while $w_L^* D(p_L^*) = w_H^* D(p_H^*)$ at $\overline{s} = 0$) it follows that $D(p^*) dw_L + w^* D'(p^*) dp_L < D(p^*) dw_H + w^* D'(p^*) dp_H$ so that $dw_H - dw_L > 2\,(dp_H - dp_L)$ and continue using the proof of Proposition 7.